

Dynamic evaluation of an Arch Dam-Foundation-Reservoir system considering randomly fluctuating material properties within the foundation and the dam

Maroua Hammami^{1,2}, Régis Cottureau¹

¹ MSSMAT UMR 8597, CentraleSupélec, CNRS, 92290, Châtenay Malabry, France, {maroua.hammami,regis.cottureau}@centralesupelec.fr

² Tractebel Engie, 5 Rue du 19 Mars 1962, 92622 Gennevilliers, France

Abstract — A concrete arch dam has been analysed during seismic loading with a model based on spectral elements. We used an hexahedral mesh generated by the free 3D finite element grid generator Gmsh. In this contribution, the dam and the soil are, firstly, considered as isotropic and homogeneous. Then, material parameters are modeled as random fields in order to study the structural response. Finally, an interpretation and a comparison of the results in terms of eigenfrequencies and mode-shapes is given to investigate the impact of heterogeneities within the foundation and the dam on the structural response.

Mots clefs — seismic response, arch dam, heterogeneities.

1. Introduction

The evaluation of seismic response of soils and structures constitutes an important problem in relation to the ground motion amplification. There always exist uncertainties in defining the properties of soils. This results from natural heterogeneity or the variability of the soil and limited availability of information about material properties. In particular, using well-log data collected in various areas of the world, several authors (Wu et al. 1994, Kneib 1995, Holliger 1996, Shiomi et al. 1997) have constructed random models for these fields of mechanical properties, including correlation models between P- and S-wave velocities (Birch 1961). Most of these authors measure a relative standard deviation close to 5%, but the consensus is not so clear about the correlation length, which is measured between 1 m and 100 m. In addition to this direct evidence based on well-log data, hundreds of travel-time tomography campaigns over the years have proven that the crust is heterogeneous on scales of 1 km to 10 km (Schilt et al. 1979, Aki and Lee 1976, Aki et al. 1976, Zhang and Thurber 2003, Zhao et al. 2009, Takemura et al. 2015). In geotechnics, cone penetrometer tests (Fenton 1999) and spectral analysis of surface waves tests (Schevenels et al. 2008) have identified correlation lengths of the order of 1 m in the vertical direction and of 100 m in the horizontal direction (Phoon and Kulhawy (1999)). According to these published research, heterogeneities in both structures and their environment call for the use of probabilistic approaches in order to provide a reliable design of the structure. Thus, a heterogeneous medium can be modelled as a domain with randomly fluctuating material properties.

This work aims at evaluating the influence of heterogeneities (within the foundation and the arch dam) in an arch dam response under a seismic loading. The mass of the foundation is taken into account and a seismic source is introduced. The reservoir is considered full of water. The results presented in this paper are the eigenfrequencies and mode shapes over a particular frequency band. A dam-reservoir is, firstly, coupled to a homogeneous soil. Then we consider heterogeneity in both dam and foundation. A special attention is paid to compare the results in those different hypotheses.

2. Numerical modelling

2.1. Constitutive equations

The equilibrium equation is obtained by expressing that the sum of the rate of work by external forces and the rate of work by internal forces is equal to the rate of work by quantities of acceleration, that is (1) :

$$\operatorname{div} \sigma(U) + f = \rho \frac{\partial^2 U}{\partial t^2}(x, t) \quad (1)$$

where σ is the Cauchy stress tensor, f the body forces, ρ the density and U the displacement. The isotropic linear elastic constitutive law in small strains is given by the Hooke's law:

$$\sigma(U) = \lambda \operatorname{div}(U) Id + 2\mu \epsilon(U) \quad (2)$$

where λ and μ denote the Lamé constants, Id the identity 2nd order tensor and ϵ the strain tensor.

In consequence, it is always possible to decompose the total displacement field U as the sum of a gradient of a scalar potential Φ and the curl of a vectorial potential Ψ :

$$U(x, t) = \nabla \Phi(x, t) + \operatorname{rot} \Psi(x, t) \quad (3)$$

Solving the equilibrium equation in an homogeneous medium taking into account equations (2) and (3), we obtain the following equations for the potentialals:

$$\begin{cases} \rho \frac{\partial^2 \Phi}{\partial t^2} - (\lambda + 2\mu) \Delta \Phi = 0 \\ \rho \frac{\partial^2 \Psi}{\partial t^2} - \mu \Delta \Psi = 0 \end{cases} \quad (4)$$

in which we consider $C_p = \sqrt{\frac{\lambda+2\mu}{\rho}}$ and $C_s = \sqrt{\frac{\mu}{\rho}}$ that denote respectively the pressure wave velocity and the shear wave velocity.

2.2. Spectral Formulation

The spectral element method is a finite element method with a high-order of interpolation to guarantee the spectral convergence (Cohen 2002, Priolo 1994, Faccioli 1997, Komatitsch 2005, Seriani 1998 et Komatitsch 1999).

The displacement field U is given in each element on a base of Lagrange polynomials at order N . These polynomials are defined on the points of Gauss Lobatto-Legendre (GLL).

$$u(x, y, z) = \sum_{i,j,k} U_{i,j,k} \phi_i(x) \phi_j(y) \phi_k(z) \quad (5)$$

One of the originalities of the method, results from the choice of the quadrature for the numerical evaluation of the finite element formulation. Gauss points are taken astutely the same as the nodes of definition of the basic functions. Therefore, the mass matrix M is diagonal (easy to inverse).

$$M_{i,j} = \sum_k \alpha_k \phi_i(x_k) \phi_j(x_k) \quad (6)$$

$\phi_i(x_k) \phi_j(x_k) = \delta_{ij}$ where δ_{ij} is the Kronecker symbol.

2.3. Time integration method

We discretize the time interval of interest using a time step Δt . The discrete momentum equation is then enforced in conservative form at $t_{n+\frac{1}{2}}$:

$$\frac{1}{\Delta t} M [V_{n+1} - V_n] = F_{n+\frac{1}{2}}^{ext} - F^{int}(U_{n+\frac{1}{2}}^h, V_{n+\frac{1}{2}}^h) \quad (7)$$

$$U_{n+1} = U_n + \frac{\Delta t}{2} (V_n + V_{n+1}) \quad (8)$$

$$A_{n+1} = \frac{(V_{n+1} - V_n)}{\Delta t} \quad (9)$$

where $U_{n+\frac{1}{2}} = \frac{(U_{n+1} + U_n)}{2}$, $F_{n+\frac{1}{2}}^{ext} = \frac{(F_{n+1}^{ext} + F_n^{ext})}{2}$, M , F^{int} , F^{ext} , U , V and A denote respectively the mass matrix, the internal forces, the external forces, the displacement, the velocity and the acceleration.

Unlike in the implicit scheme, there is no need to assemble and invert the global mass matrix. The time step Δt has to be smaller than the critical time step Δt_{cr} which in an undamped system depends on the highest frequency in the smallest element (Komatitsch et al. 1999).

3. Modal analysis of an arch dam coupled to a homogeneous foundation

This work consists on the dynamic analysis of an arch dam under seismic loading using the SEM (Spectral Element Method) software. Details of the procedures utilized, the results of the computations and an interpretation on the results are presented in the following sections.

3.1. Presentation of the model

The mesh was generated by the free 3D finite element grid generator Gmsh . It contains 42117 nodes and 37360 hexahedral elements.

Material properties are listed in the table 1 :

Tableau 1 : Material properties

	Dimension (m)	Cp (m/s)	Cs (m/s)	Density (kg/m3)
Foundation	10000X10000X6000	3554	2175	2200
Dam	200(height) 200 (lenght on the crest) 60 (lenght in the base)	3472	2195	2400
Reservoir	200 (completely full) 5000 (lenght)	1500	-	1000
Bedrock	10000X10000X6000	4500	2470	2600

The bedrock is placed in the bottom of the model (just under the foundation).

As a seismic source, we use a Ricker pulse centred at 1Hz in the middle of the bedrock. The amplitude of the signal is given by :

$$A(t) = (1 - \frac{(t-t_0)^2}{2\sigma^2})e^{-\frac{(t-t_0)^2}{2\sigma^2}} \quad (10)$$

where the parameters t_0 and σ describe the duration of this wavelet signal.

3.2. Results

In order to analyze the behavior of the dam under the seismic load, time responses are converted in the frequency domain by performing a Fourier Transform of the time signal. We choose 41 equidistant points on the dam's crest. A point in the interface dam-foundation is considered as a reference station (see figure(1)).

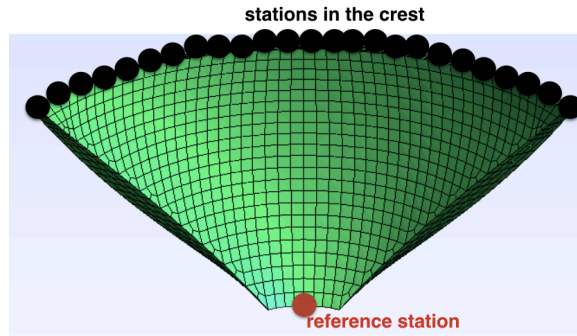


Figure 1 : Arch dam's stations

For the evaluation of the first modes of the structure, we use spectrum ratio between each point and the reference station (A. Robbe et al, 2015). Then, drawing the sum of these curves, function of frequency, it is possible to select the peaks that correspond to eigenfrequencies. In this study, cross-spectrum method is used to evaluate not only eigenfrequencies but also mode's shape. Magnitude of each cross-spectrum provides an overview of the mode's shape. Magnitude of the cumulated cross-spectrum is plotted on figure 2.

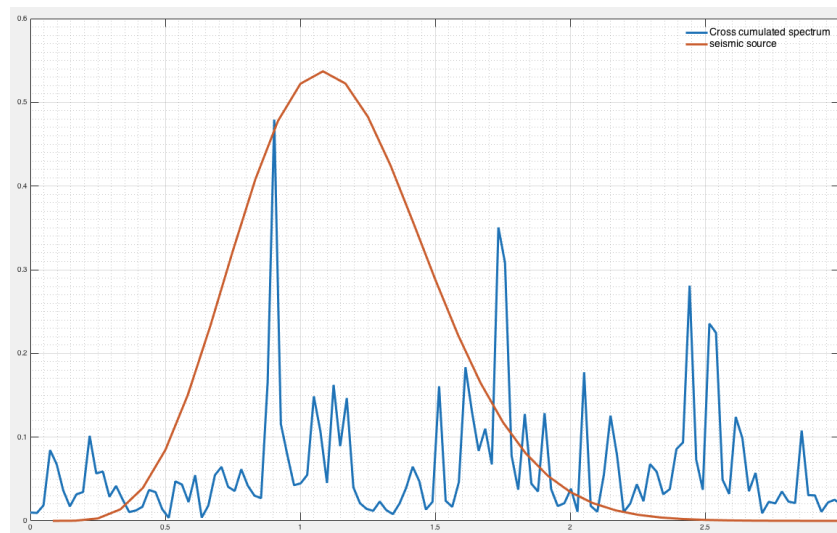


Figure 2 : Cumulated cross-spectrum (homogeneous model)

Looking at the magnitude curve, we notice many pics from which 4 most pronounced ones, in the interval [0.5 Hz 2.5 Hz], that correspond to the first 4 modal frequencies (0.9038 Hz, 1.124 Hz, 1.612 Hz and 2.443 Hz). Mode shapes are plotted in the figure (3).



Figure 3 : First 4 mode shapes of the crest

4. Modal analysis of an arch dam considering randomly fluctuating material properties

To quantify the influence of heterogeneity in the medium, probabilist mechanical parameters can be used. The spectral representation is a classic way to sample gaussian random field (G. Deodatis, M. Shinozuka 1991).

$$S_k(x) = \int_{k \in W} \hat{R}_k^{1/2} e^{ikx} dW_k \quad (x \in W) \quad (11)$$

where S is the random field, \hat{R} the Fourier transform of a correlation function R and dW the Brownian motion. In our model, S could be the density or the Lamé coefficients. The evaluation of the response variability due to system stochasticity consists of performing the response analysis of structural systems with gaussian correlation in their material properties. For this reason, in our study, we consider a stochastic processes with a correlation length equal to the minimum wavelength. Moreover, a Log-normal distribution is used for the material properties. We consider a standard deviation of 50 % for the log-normally distributed medium density and Lamé coefficients in the foundation and 12% in the dam. The correlation length is equal to 300m in the foundation and 20m in the dam.

As for the homogeneous case, we consider the same 41 equidistant points on the dam's crest. We calculate the spectrum ratio between each point and the reference station. Then, we plot the sum of these spectrum ratios, function of frequency in order to obtain the peaks that correspond to eigenfrequencies (see figure 4).

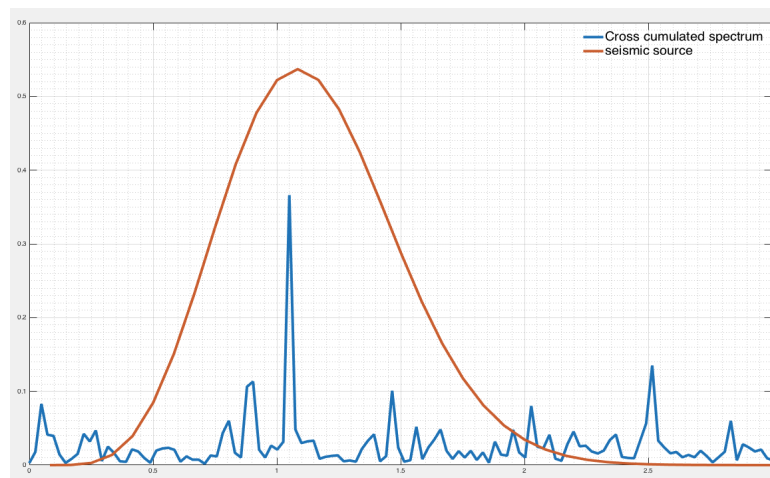


Figure 4 : Cumulated cross spectrum

Looking at the magnitude curve in the figure 4, we notice many pics from which 4 most pronounced ones, in the interval [0.5 Hz 2.6 Hz], that correspond to the first 4 modal frequencies (0.9038 Hz, 1.05 Hz, 1.466 Hz and 2.516 Hz). Mode shapes are plotted in the figure (5).



Figure 5 : First 4 mode shapes of the crest

To understand the influence of the heterogeneities on the dam-foundation-reservoir interaction, we summarize the different results obtained in the table bellow.

Tableau 2 : Comparaison of the results

	Mode 1	Mode 2	Mode 3	Mode 4
Homogeneous	0.9038 Hz	1.124 Hz	1.612 Hz	2.443 Hz
Heterogeneous	0.9038 Hz	1.05 Hz	1.466 Hz	2.516 Hz

According to the results summarized in the table 2, we note that heterogeneities does not affect the fondamental mode (mode 1). We have the same result in both models (homogeneous and heterogeneous). For the second and third modes, eigenfrequencies are lower in the case of randomly fluctuating material properties for both dam and foundation than the homogeneous media. That highlights the influence of soil and concrete heterogeneities in the structures response.

Comparing the curves of cumulated cross spectrum in the figures 2 and 4, we note that the amplitude is much lower in the heterogeneous case.

That highlights the influence of soil and concrete heterogeneities in the structures response.

5. Conclusion

The results of these analysis confirmed that considering randomly fluctuating material properties within the foundation and the dam influences the site response and could change the mechanical response of the dam. In this order of idea, we should precise that the numerical tests have to be pushed further by evaluating the influence of heterogeneities on the seismic response of arch dams considering different reservoir levels.

2.3. References

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