

An enrichment-based approach for the simulation of fretting problems

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Abstract — In this work a new numerical strategy will be assessed to evaluate the fretting problem under cylindrical contact conditions. For that purpose, a methodology introduced recently to describe the mechanical fields in the proximity of the contact edges will be used to enrich fretting simulations using the X-FEM. This strategy is expected to represent a breakthrough concerning the solution of this kind of contact problems, once that it permits to perform fretting simulations with coarser meshes facilitating the implementation in an industrial context.

Key words — Fretting, X-FEM, Model Reduction.

1. Introduction

The problem of fretting happens when we have mechanical parts in contact experiencing tangential oscillatory displacements of small amplitude (the order of μm). This kind of problems are responsible for high levels of stress concentration close to the contact surfaces which associated with the wear makes cracks very likely to occur. Things can be even worse when bulk fatigue loads are present leading to a premature failure of the components [1, 2]. A classic example where fretting fatigue takes place is the connection between blades and discs in aeronautical compressors. In this case, the centrifugal load experienced by the pads in association with the vibration loads, due to the interactions between the blades and the airflow, give origin to the fretting fatigue phenomenon.

The fretting problem is challenging in an industrial context where analytical solutions are seldom available, making FE simulations with fine meshes needed to capture the stress concentration around the contact surfaces. It makes the problem extremely expensive. In this work, a new methodology to describe the mechanical fields close to the contact edges through a crack analogy approach [3] will be used to enrich fretting simulations performed on coarse meshes. The enrichment technique used here is the X-FEM, taking advantage of the similarity found to describe the mechanical fields around the contact edges in fretting problems with the ones found in linear elastic fracture mechanics problems around the crack tip. A nonlocal approach to estimates precisely the position of the contact edge will also be proposed and assessed, once that, as we perform simulations with coarse meshes, the right position of the contact edges starts to be unknown.

2. Overview of the method

2.1. Crack analogy approach

Recently, Montebello et al. proposed a new methodology to describe the mechanical fields in fretting problems [3]. They have done an analogy to fracture mechanics problems and described the velocity field close to the contact edges through separated variables. The idea consisted in describing the local velocity field in a referential attached at the contact edges as product of some nonlocal intensity factors, that are capable to capture the external loads effects, and some local spatial reference fields, that are

able to catch the local geometric effects of the problem. The velocity field can then be expressed as follows:

$$\mathbf{v}_{R'}(\mathbf{x}, t) = \dot{I}^s(t)\mathbf{d}^s(\mathbf{x}) + \dot{I}^a(t)\mathbf{d}^a(\mathbf{x}) + \dot{I}^c(t)\mathbf{d}^c(\mathbf{x}) \quad (1)$$

where we first need to compute the symmetric and antisymmetric parts of the spatial reference fields \mathbf{d}^s and \mathbf{d}^a , respectively. First, it is needed to show the problem to be solved and the load history applied to perform the decomposition of the velocity field. The problem solved is the cylindrical contact configuration shown in Figure 1a. Firstly, a normal load P is applied on the pad pressing it against the specimen that is fixed on the bottom and on both lateral sides (note the small perturbation in the normal load before its final plateau). Secondly, a tangential oscillatory load Q is applied on the pad imposing the fretting conditions, Figure 1b. The ratio $Q/P < \mu$ is always respected, where μ is friction coefficient. It ensures the partial slip condition.

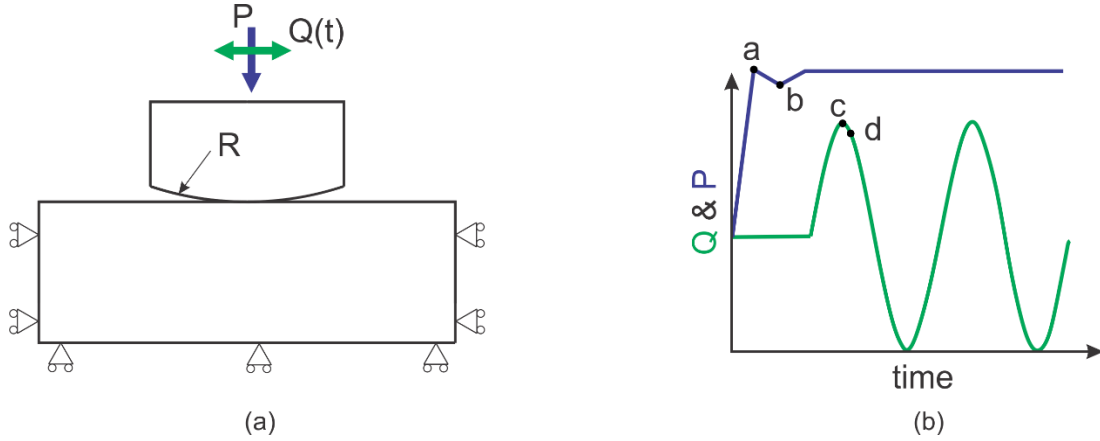


Figure 1 - (a) Cylindrical contact under fretting conditions, (b) load history applied to extract the spatial reference.

The spatial reference fields \mathbf{d}^s and \mathbf{d}^a can be computed extracting the velocity field in some strategic points of the load history, Figure 1b:

$$\mathbf{d}^s(\mathbf{x}) = \frac{\mathbf{x}(t_b) - \mathbf{x}(t_a)}{t_b - t_a}, \quad \mathbf{d}^a(\mathbf{x}) = \frac{\mathbf{x}(t_d) - \mathbf{x}(t_c)}{t_d - t_c} \quad (2)$$

The idea is to catch separately the contributions of the normal load and the tangential load to describe the mechanical fields. It is worth mentioning that when these fields are computed the whole contact is in stick condition, therefore, both bodies behave as only one and the nonlinear effects inside the slip zones are not taken into account. In this case, the problem is very close to a fracture mechanics problem where the contact edge behaves like the crack tip in fracture mechanics problems. The normal load behaves as a load in mode I and the tangential load behaves as a load in mode II in fracture mechanics problems. This is the reason why \mathbf{d}^s and \mathbf{d}^a are named like this, symmetric and antisymmetric part, respectively. The nonlocal intensity factors \dot{I}^s and \dot{I}^a can be computed projecting the actual velocity field, extracted from fine FE simulations, on the basis \mathbf{d}^s and \mathbf{d}^a :

$$\dot{I}^s(t) = \int_{\Omega} \mathbf{v} \cdot \mathbf{d}^s d\Omega \Big/ \int_{\Omega} \mathbf{d}^s \cdot \mathbf{d}^s d\Omega, \quad \dot{I}^a(t) = \int_{\Omega} \mathbf{v} \cdot \mathbf{d}^a d\Omega \Big/ \int_{\Omega} \mathbf{d}^a \cdot \mathbf{d}^a d\Omega \quad (3)$$

Now the linear part of the velocity field can be defined as follows:

$$\mathbf{v}_e = \dot{I}^s(t)\mathbf{d}^s(\mathbf{x}) + \dot{I}^a(t)\mathbf{d}^a(\mathbf{x}) \quad (4)$$

The spatial reference fields \mathbf{d}^s and \mathbf{d}^a can be expressed in polar coordinates applying a POD to these fields:

$$\mathbf{d}^s(x) \cong f^s(r)\mathbf{g}^s(\theta), \quad \mathbf{d}^a(x) \cong f^a(r)\mathbf{g}^a(\theta) \quad (5)$$

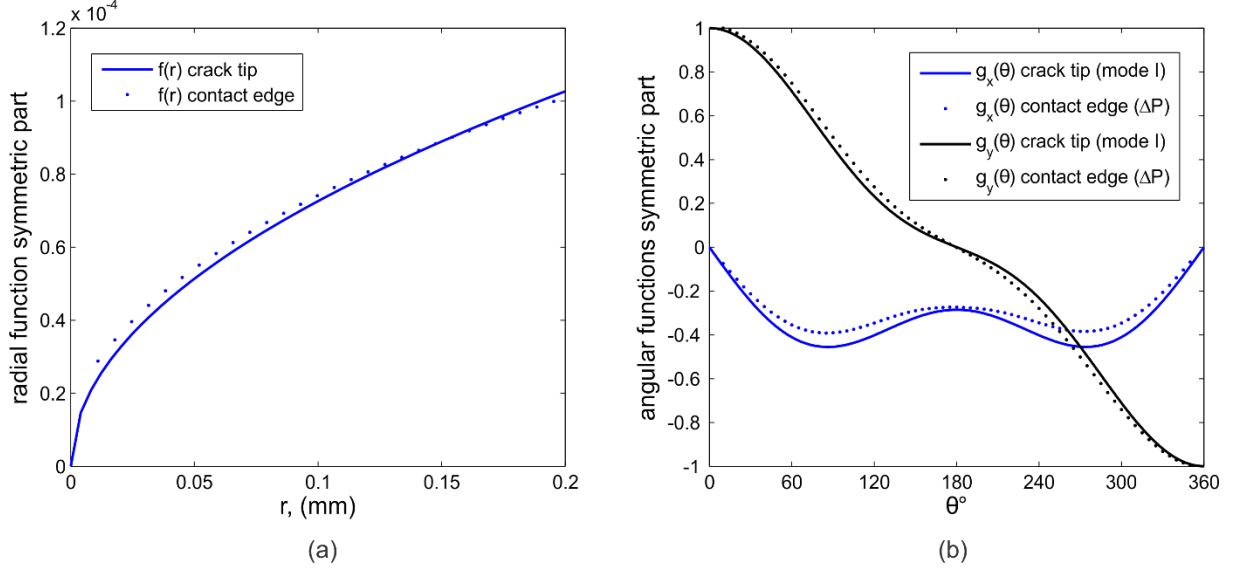


Figure 2 - (a) Comparison between the radial evolutions of \mathbf{d}^s and the radial evolution of the displacement field of a crack in mode I, (b) comparison between the tangential evolution of \mathbf{d}^s and the tangential evolution of the displacement field of a crack in mode I.

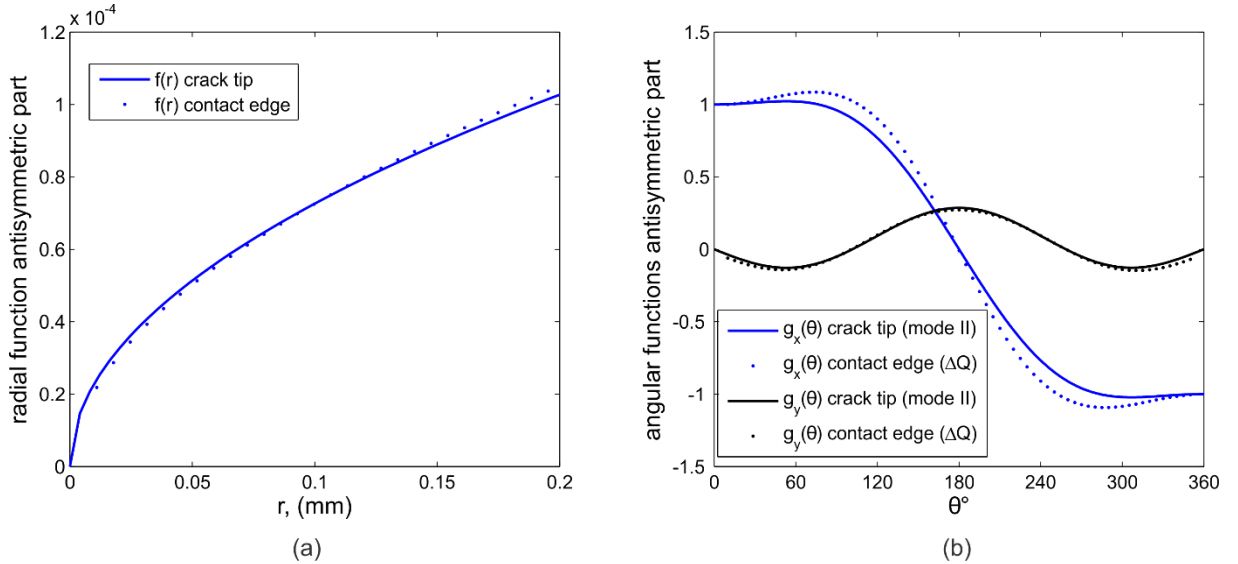


Figure 3 - (a) Comparison between the radial evolutions of \mathbf{d}^a and the radial evolution of the displacement field of a crack in mode II, (b) comparison between the tangential evolution of \mathbf{d}^a and the tangential evolution of the displacement field of a crack in mode II.

In Figures 2 and 3, these polar functions, used to describe the spatial reference fields under fretting conditions, are compared with the ones found to describe the mechanical fields close to the crack tip in linear elastic fracture mechanics (LEFM) problems. The similarity of these fields, depicted in Figures 2 and 3, confirms the crack analogy approach proposed by [3] and encourages us to take advantage of this

behaviour to try to enrich fretting simulations performed using coarse meshes. It is possible to improve the accuracy to describe the velocity field adding a complementary part term to the description of the velocity field. This complementary term could be obtained computing the residual error of the linear approximation, expressed by the actual velocity field minus the linear approximation. However, if we wanted to consider the complementary part of the velocity field to enrich our simulations, it would be necessary a handful of modes (pairs of functions) in order to describe the spatial reference field of the complementary part of the velocity field, \mathbf{d}^c , in polar coordinates. At least 5 modes are needed as shown in Figure 4a. It is also worth mentioning that the first mode describing this field (the most important one) has a radial exponential behaviour, as shown in Figure 4b, which means that the effects of this field are confined in a region close to the contact edges. Therefore, henceforward, only the spatial reference fields due to the linear part of the velocity field will be used to enrich the fretting simulations.

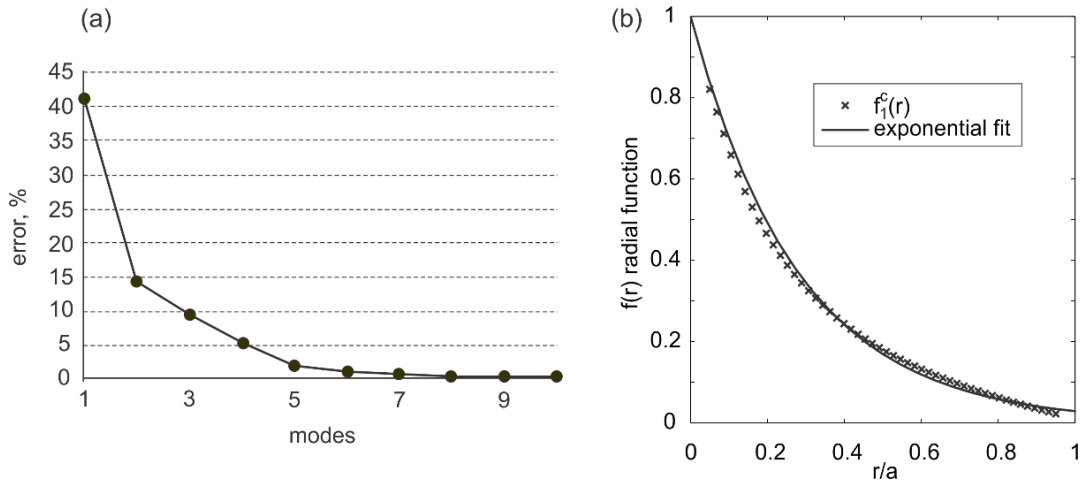


Figure 4 - (a) Residual error to describe \mathbf{d}^c , (b) radial evolution of the first mode to describe \mathbf{d}^c .

2.2. The X-FEM applied to solve fretting problems

The problematic here is to capture the strong stress concentration around the contact edges in fretting problems using coarse meshes. To accomplish that the FE approximation will be enriched using the partition of unity framework as previously applied in LEFM [4]. In the standard FEM, the displacement field can be approximated and expressed as:

$$\mathbf{u}(\mathbf{x}) = \sum_{i \in I} N_i(\mathbf{x}) \mathbf{u}_i \quad (6)$$

where I is the set of all nodes discretizing the domain, \mathbf{u}_i is the displacement field at the node i and N_i is corresponding basis function of this node. It is well known that the FE basis function represents the partition of unity. The basic idea behind the X-FEM is the multiplication of the nodal basis functions $N_i(\mathbf{x})$ with some enrichment functions $\psi(\mathbf{x})$. Defining J as the subset of enriched nodes, $J \subset I$, the enriched approximation using the partition of unity can be expressed as:

$$\mathbf{u}(\mathbf{x}) = \sum_{i \in I} N_i(\mathbf{x}) \mathbf{u}_i + \sum_{j \in J} N_j(\mathbf{x}) \sum_{\alpha} \psi_{\alpha}(\mathbf{x}) \mathbf{a}_{j,\alpha} \quad (7)$$

where ψ_{α} are the set of enriched functions at each enriched node $j \in J$ multiplying the new degrees of freedom $\mathbf{a}_{j,\alpha}$. This approach permits us to inherit some properties of the FE basis functions, such as their compact support, and hence preserving advantages of the standard FEM, such as the symmetry and sparsity of the stiffness matrix.

The goal here is to take advantage of the possibility to describe the mechanical fields around the contact edges in fretting problems by means of a LEFM approach. Then, the displacement field in the proximity of the contact edges may be enriched using the following analytical functions, the same ones used to capture the strong stress singularity in LEFM problems:

$$\psi_{\alpha} = \left\{ \sin \frac{\theta}{2}, \cos \frac{\theta}{2}, \sin \frac{\theta}{2} \sin \theta, \cos \frac{\theta}{2} \sin \theta \right\} \quad (8)$$

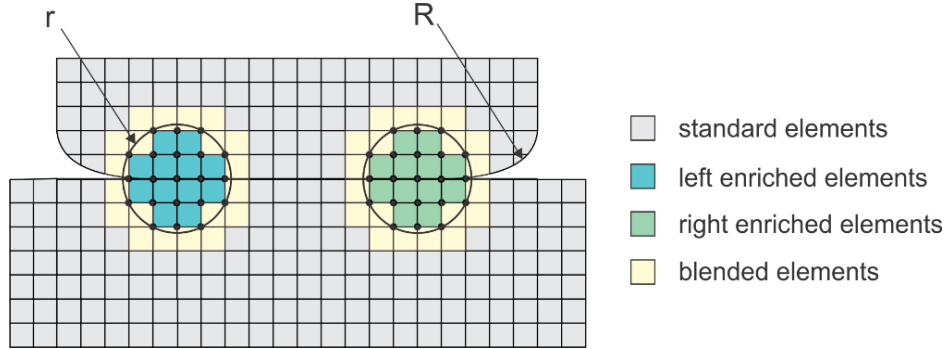


Figure 5 - Enriched scheme used to solve fretting problems.

Figure 5 depicts schematically the fretting enrichment technique applied to solve fretting problems. The lower case “r” represents the enrichment radius, which permits to define the elements that will be enriched during the simulations. As can be seen, one has a composition of enriched nodes, in the proximities of the contact edges and some blended elements (yellow elements) surrounding the fully enriched elements (blue and green elements). It is worth mentioning here that, in order to apply the enrichments to this problem, the position of the contact edges were assumed to be known, either due to previous simulation performed on fine meshes or due to analytic solutions.

3. Results

To check if the enrichment technique in association with the crack analogy approach are capable to improve the results of fretting simulations on coarse meshes, the problem shown in Figure 6a will be solved. It is worth mentioning that this cylindrical contact configuration under partial slip regime admits analytical solution. Hence, the analytical solution of the studied problem with both standard and enriched FE simulations will be compared.

The load history applied to this problem is depicted in Figure 6b. In this case, first a vertical displacement, u_y , is applied on the pad pressing it against the rectangular specimen, which is fixed on the bottom. After that, a tangential sinusoidal displacement, u_x , is applied on the pad imposing the fretting conditions. All the simulations were performed considering a quasi-static elastic behaviour. The young modulus, E , used is 200 GPa and the Poisson ration, ν , is 0.3. The pad and the specimen have the same material properties. The relation $Q/P < \mu$ is respected during the whole load history, where Q is the total tangential load and P is the total normal load between the contact surfaces, ensuring that no gross slip is observed.

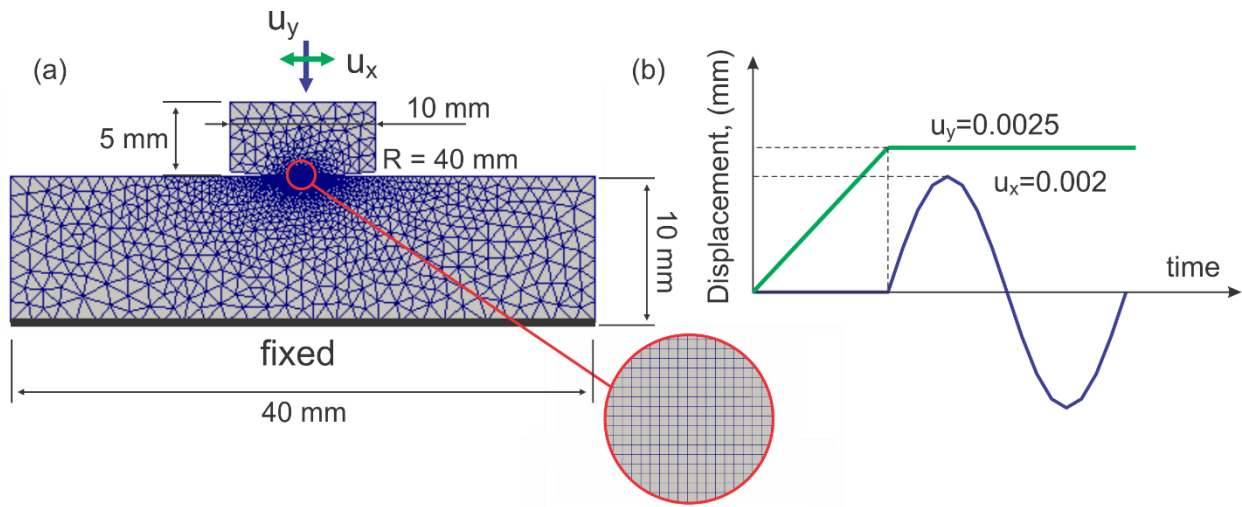


Figure 6 – (a) Fretting problem model, (b) Load history applied.

The simulations were run considering structured rectangular linear elements near the contact surfaces. The contact problem was solved using the LATIN method [5], considering the Coulomb's law with a friction coefficient of 0.9. The LATIN method is not the core of this work, but this technique has already been used to compute damping due to friction in joints [6] and solve fretting problems [7]. After the simulations the stress evolution in two different directions were analysed. Firstly, the stress evolution was evaluated at a fixed position on the contact surface, left edge, while we moved away vertically from the contact edge inwards the specimen, Figure 7. Secondly, the stress evolution was assessed along the contact surfaces, Figure 8. To verify the gain introduced by the fretting enrichments, three simulations were performed. The first one considers a standard FE simulation with a coarse mesh, with only 18 elements discretizing the contact region (0.025 mm). The second one considering the same mesh used in the first simulation, but in this case the fretting enrichments were applied to the problem. The third simulation was performed using the standard FE method, but discretizing the problem with a fine mesh, around 88 contact elements (0.005 mm), which allows us to obtain results very close to the analytical ones.

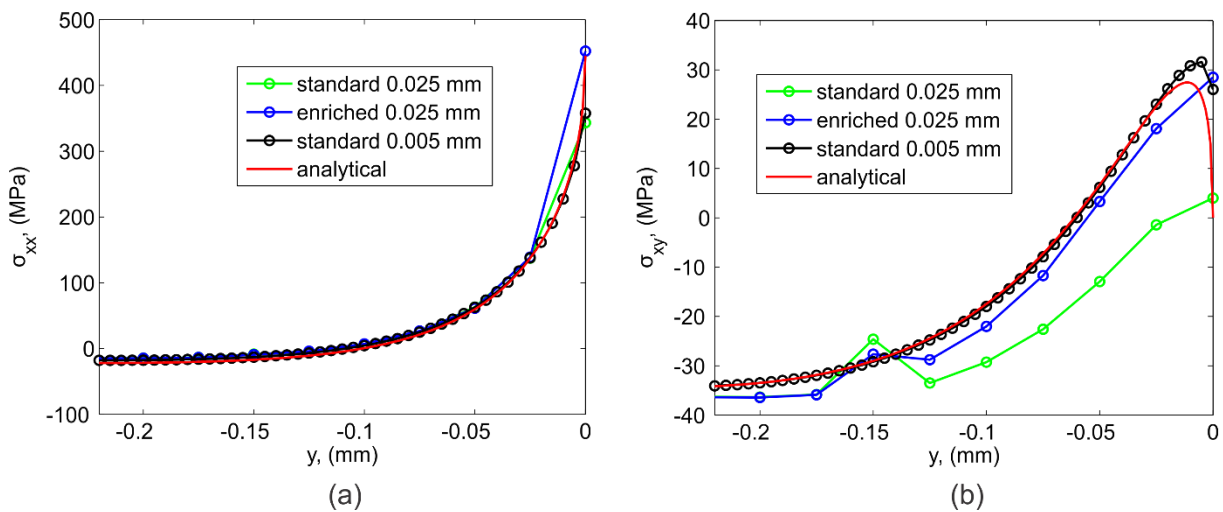


Figure 7 - Stress evolution vertically inwards the specimen: (a) σ_{xx} stress component, (b) σ_{xy} stress component.

It is possible to see that whenever the enrichments are present, the results provided by the simulations

are better, Figures 7 and 8. Particularly, when we analyse the σ_{xx} stress component in the vertical direction, Figure 7a, it is possible to see that even using a coarse mesh the enriched simulation has a very good accordance with the analytical solution (red curve). Whereas, when we analyse the σ_{xy} stress component, Figure 8b, one can see a high improvement in the results when the enrichments are used. In this case, the solution is almost as accurate as the one obtained using a mesh 5 times smaller (0.005 mm).

All simulations were performed considering an enrichment radius of $0.6a$. Figure 9 depicts the influence of the enrichment radius on the simulations, where different simulations were run keeping the same mesh and load conditions but changing the enrichment radius. What can be seen is that for enrichment radius higher than $0.4a$ the results remain almost the same. It is good once that there is no need to enrich large areas around the contact edges.

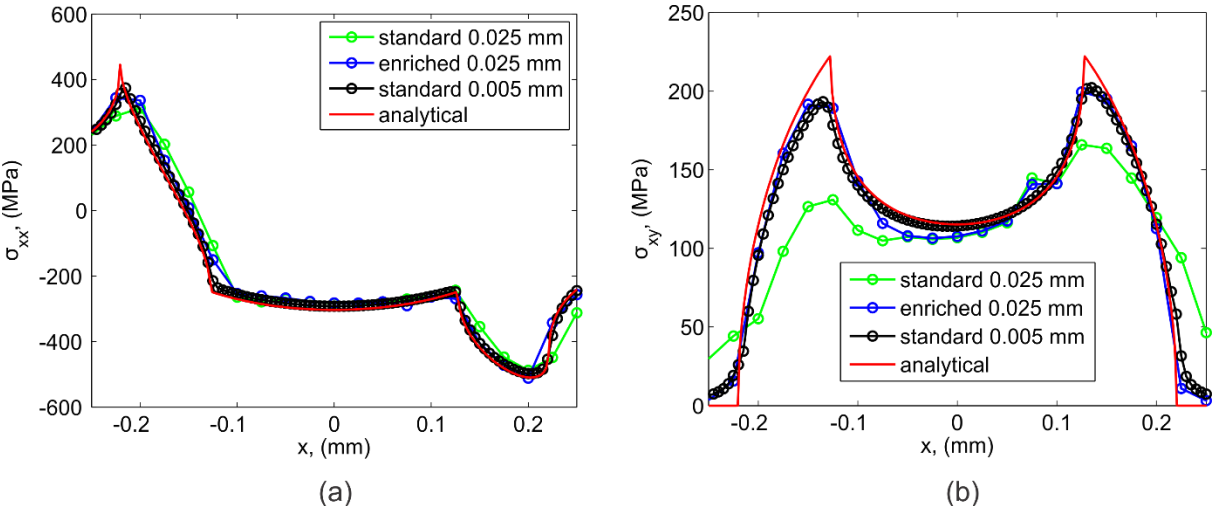


Figure 8 - Stress evolution along the contact surface: (a) σ_{xx} stress component, (b) σ_{xy} stress component.

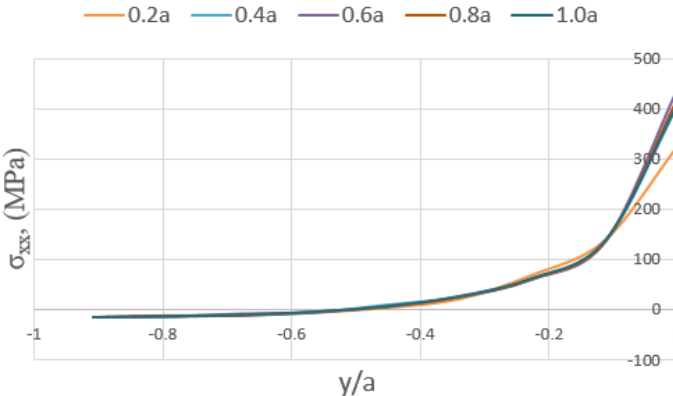


Figure 9 - Enrichment radius influence (mesh size at the contact zones 0.025 mm).

4. Preliminary conclusions

Fretting simulations under cylindrical contact conditions were enriched aiming to increase the quality of the FE solutions performed on coarse meshes. The enrichment functions were chosen by means of a

crack analogy approach. To enrich the simulations the X-FEM was used. The idea of this kind of approach is to reduce the computational costs to solve these types of local problems. The first results seem promising once that the quality of the FE solutions after introduce the enrichments functions were notably increased. Further studies must be done to keep verifying the advantages and drawbacks of the use of this type of approach to enrich fretting simulations. Besides, a new nonlocal approach to estimate precisely the position of the contact edge should be developed, once that, this task becomes increasingly difficult as we start to work with coarse meshes.

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