CSMA 2017
13ème Colloque National en Calcul des Structures
15-19 Mai 2017, Presqu'île de Giens (Var)

Wave propagation in a stochastic media: dispersion curves for a ballasted railway track

L. de Abreu Corrêa[1], R. Cottereau[1], C. Voivret[2],
S. Costa d'Aguiar[2], E. Bongini[2], B. Faure[2]

[1] *Laboratoire MSSMat UMR 8579, CentraleSupélec, CNRS, France {lucio.de-abreu-correa, regis.cottereau}@centralesupelec.fr*
[2] *SNCF, Direction de l'Innovation et de la Recherche, France, {charles.voivret, sofia.costadaguiar, estelle.bongini, baldrik.faure}@sncf.fr*

Abstract — This paper discusses the dynamical behavior of a randomly-fluctuating heterogeneous continuum model of the ballast. The Young's modulus is modeled as a random field parameterized by its average, its variance and a correlation model. A numerical model of the ballast and the surrounding soil is then constructed based on an explicit spectral element solver. This model allows to describe numerically the wave field generated, as well as to construct dispersion relation for the ballast-soil model. The influence of heterogeneity is discussed by comparison with a similar homogeneous model.
Keywords — Wave propagation, Stochastic media , Railway track, Dispersion Curves.

1 Introduction

The competition with other means of transportation has increased the demand for performance in the railway industry. The dynamical loads caused by the passage of high-speed trains accelerate track deterioration and damage neighbor buildings [1]. In order to study this problem two classes of numerical models are used to try predict this behavior: (1) discrete approaches, in which each grain of the ballast is represented by a rigid body and interacts with its neighbors through nonlinear contact forces (using molecular dynamics or non-smooth contact dynamics); and (2) continuum approaches, in which the ballast is replaced by a homogenized continuum and the classical Finite Element Method (FEM, or similar) can be used to solve the problem. The discrete approaches are today capable of solving only a few meters-length of ballast. The absence of an efficient scheme for parallelization over large clusters of computers [2] and the coupling between the grains and the soil [3] remains an open problems. On the other hand, the methods based on continuum mechanics (like FEM) can solve very large models thanks to efficient parallelization schemes. However, a homogeneous continuum cannot represent the complex fields of stress and strain present in the actual mechanical system.

The dynamical behavior of ballast can be completely described by its dispersion curve. The numerical simulation of dispersion curves in railway tracks is usually performed on homogeneous media [4, 5, 6]. The most advanced models consider 3D finite elements and model reduction [7]. Unfortunately, these homogeneous models cannot reproduce accurately the experimental observations [8]. The discrete nature of the material and the complex interaction between the grains produce a ”non trivial” mechanical behavior. It is necessary to consider the granular character of the material. The simulation of wave propagation in granular media for simple geometries has been treated using discrete models, and has succeeded in reproducing a wide range of experiments [9, 10]. The influence of pre-stress on the wave velocity in particular can be simulated [11] as well as the influence of the pressure in the grain network [12]. The simple addition of random "heterogeneity" in a spring network allows to reproduce the complex frequency response seen in measurements of sound propagation in a granular system [13]. Note that [9] observed the coexistence of an attenuated coherent signal and the "coda", which is a classical feature of seismic signals [14].

The objective of this work is use a statistical approach to model the mechanical properties of the ballast layer. This statistical approach aims to reproduce in a continuum model the heterogeneity of the stress field in a discrete granular model. The mechanical properties (in practice the Young's modulus) are modeled as stochastic random fields. This approach was introduced in [15] and we apply it here to the construction of dispersion curves of ballasted railway tracks in the frequency-wave-number domain.

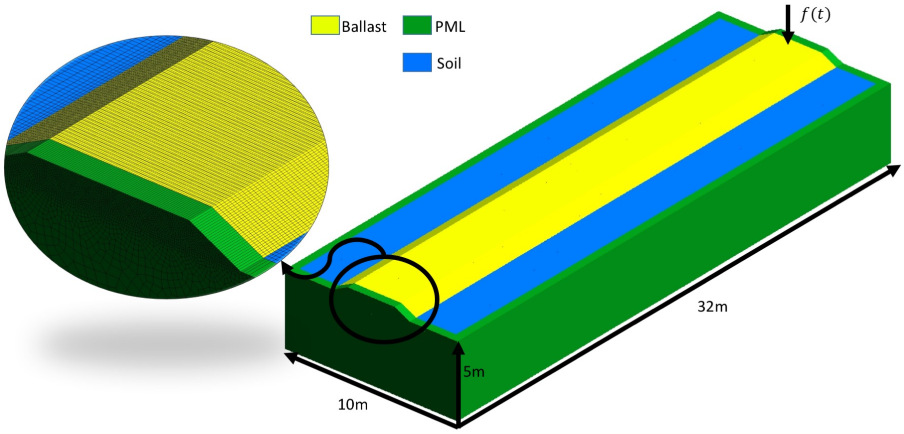


Figure 1: Geometry, mesh, and boundary conditions for the wave propagation analysis.

2 Numerical Framework

2.1 Geometry and boundary conditions

Three models were built, one of those models can be seen in the Fig.1, it was based in a 250000 elements, with 375 degree of freedoms in each element, given $\approx 94\text{x}10^6$ degrees of freedom. This model is 32 meters long. One prescribed force was impose as excitation. A rickers pulse with amplitude 1 and central frequency 30 Hz, for the other models the central frequency were modified. The Fig. 1 show also the region where vertical load is applied in the top of the ballast layer, and the mesh used. This kind of load condition was used in order to excite the "same modes" that the train usually applied in the ballast layer.

The other two models had 320000 and 380000 elements with $120\text{x}10^6$ and $142\text{x}10^6$ degrees of freedom. They only difference is the length of the railway, one is 40 and the other 48 meters long. The excitation for this models are 100 and 210Hz. Due the large number of degree of freedoms present in the model we choose a parallelizable and scalable code. This code, called SEM3D, it solve the dynamic equation in a time domain, using a explicit time integration [16, 17].

Two kind of simulations were performed in each model, homogeneous and heterogeneous for the ballast material. The statistical mean of the mechanical properties was keep the same in this two models. The Fig.2 show the position of the captures used to record the displacement field in the model. Three rows of captures were put inside of the ballast co-linear to the longitudinal axis, placed in the top, half, and the bottom of the ballast layer.

A stochastic model was used to describe the mechanical properties of the ballast. More details about this kind of model can be found in [15]. The mechanical properties used in the soil are the same of the mean properties of the ballast, the follow: $V_s = 150$m/s, $V_p = 380$m/s, and $\rho = 1500$kg/m^3.

3 Influence of the heterogeneity

3.1 Analysis of the wave patterns

Before describing the dispersion results, we present below general wave patterns obtained in the models considered. In Fig. 3, snapshots are plotted at different times for the three homogeneous models. Each line represents a different model, with the upper one corresponding to the lower frequency excitation and the lower one to the higher frequency excitation. From left to right, we observe snapshots at times $t = 0.05$ s, $t = 0.15$ s and $t = 0.25$ s, and clearly see a wave emitted at the right of the railway track propagating to the left. As the soil has the same properties as the ballast, the model is very similar to a half-space and we indeed see a wave propagating essentially as in a half-space, with very little influence of the shoulders of the track. The main difference between the three excitation frequencies is that the

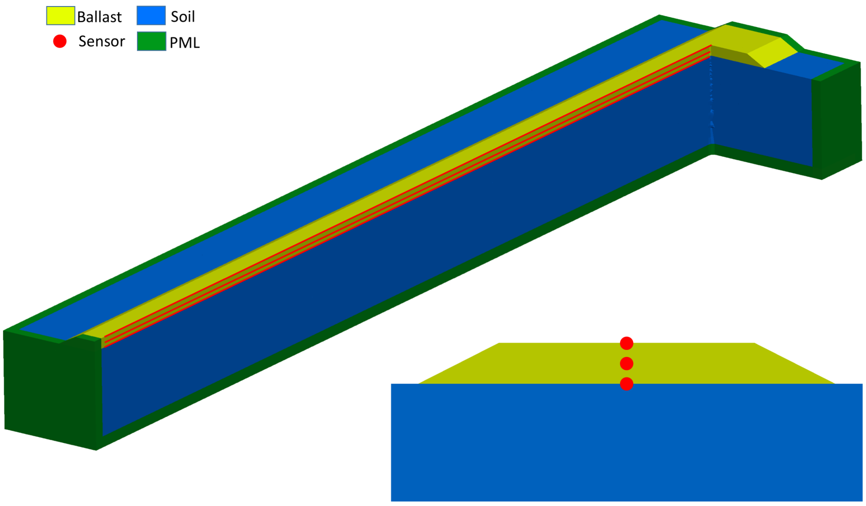
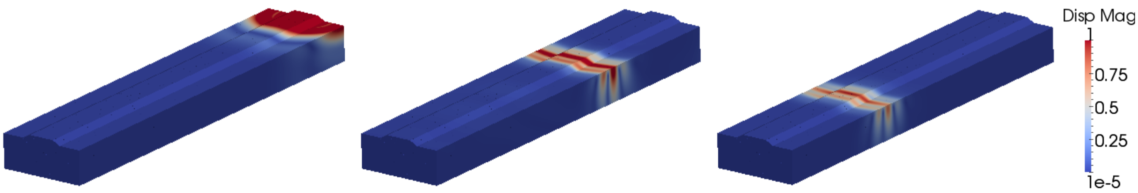
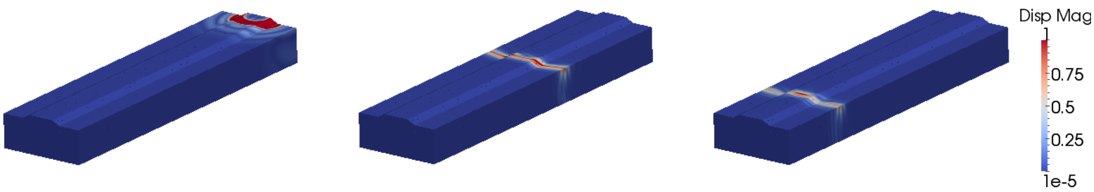


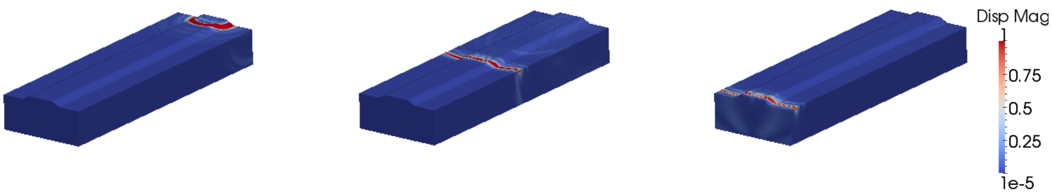
Figure 2: Cut view of the geometry and position of the captures lines. The red lines corresponds to the position of the captures lines. Three different sets where plotted at the top of the ballast, in the half of the layer, and on the interface between soil and ballast.



(a) excitation centered at 30 Hz



(b) excitation centered at 100 Hz

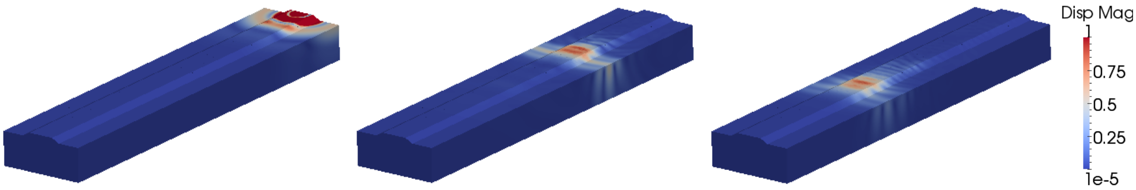


(c) excitation centered at 210 Hz

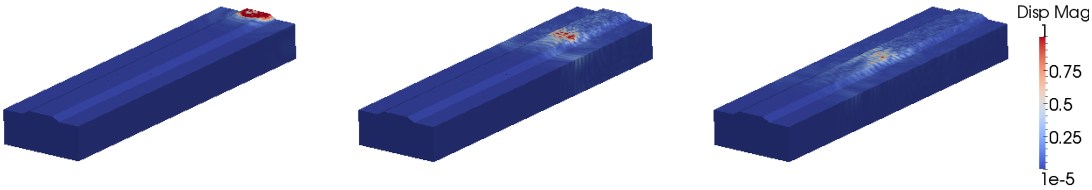
Figure 3: Displacement fields in the homogeneous model with soft soil at times $t = 0.05$ s (left column), $t = 0.15$ s (center column) and $t = 0.25$ s (right column). The different models are centered around frequencies 30 Hz (upper line), 100 Hz (central line) and 210 Hz (lower line).

propagating wave has shorter wavelength.

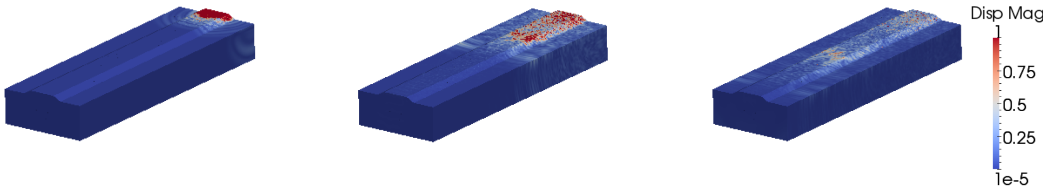
We now consider the simulations in a heterogeneous model, where the soil is homogeneous and the ballast is heterogeneous. They are plotted in Fig. 4, and several comments can be made. At lower frequencies, the energy still propagates, but there is a clear concentration of the amplitudes within the ballast layer. Although the average values of the mechanical parameters in the ballast are the same as the values in the soil, the overall effect of the heterogeneity is to create a mismatch that retains the energy within the ballast. At higher frequencies, this effect increases and is completed by the creation of a strong coda. This coda is seen as a transformation of the coherent energy in the main pulse into a trailing incoherent energy widely dispersed behind that pulse. Finally at the largest frequency, part of the coda is not even propagating, remaining right below the input energy. This phenomenon is called strong



(a) excitation centered at 30 Hz



(b) excitation centered at 100 Hz



(c) excitation centered at 210 Hz

Figure 4: Displacement fields in the heterogeneous model with soft soil at times $t = 0.05$ s (left column), $t = 0.15$ s (center column) and $t = 0.25$ s (right column). The different models are centered around frequencies 30 Hz (upper line), 100 Hz (central line) and 210 Hz (lower line).

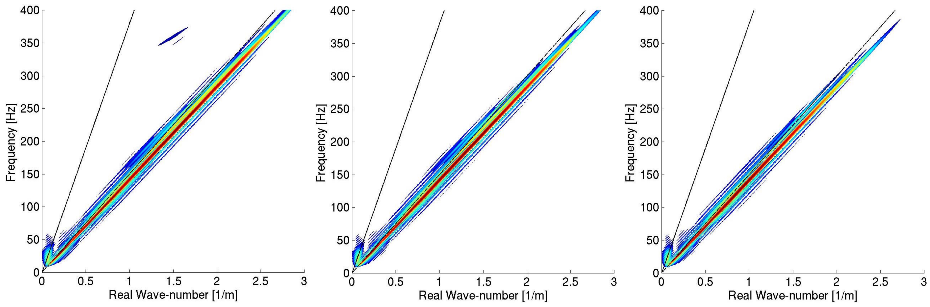


Figure 5: Dispersion curves for the heterogeneous ballast resting on soft soil, in three positions within the section of the ballast. Dispersion relations for the shear wave (black dotted line) and pressure wave (black solid line) in the ballast and soil are also represented.

localization [10], and is a direct consequence of the random heterogeneity of the ballast properties.

3.2 Analysis of the dispersion curves

We start with the homogeneous model of the ballast, with properties matching those of the underlying soil. The dispersion curves are plotted in Fig. 5. As expected, we retrieve approximately the dispersion relation for a Rayleigh wave in a homogeneous medium, with a velocity close to that of a shear wave (indicated by the black dotted line on Fig. 5). Although most of the displacement field corresponds to shear along the line that we are measuring, a small amount of P wave is also encountered at extremely low frequency, creating coupling between P and S waves around the excitation point.

The dispersion relations corresponding to the heterogeneous ballast are plotted in Fig. 6. Two important effects can be observed: (i) the propagating behavior is dispersive in the low frequency range, with a very strong slowing of the wave with increasing frequency, similar to that observed in the experimental work of [18] on unconsolidated granular packings, and (ii) the dispersion relation vanishes at higher frequencies, which is compatible with the localization effect discussed in the previous section. Note that it does not mean that there is no energy in the system, but only that the energy does not propagate (or has

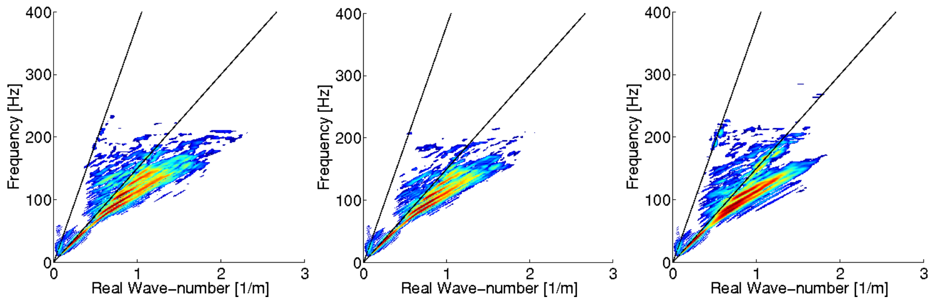


Figure 6: Dispersion curves for the heterogeneous ballast resting on soft soil, in three positions within the section of the ballast. Dispersion relations for the shear wave (black dotted line) and pressure wave (black solid line) in the ballast and soil are also represented.

a very low energy velocity). We also seem to observe some energy along the P-wave dispersion curve, which is probably due to a strong coupling of P and S wave on the heterogeneities of the ballast.

4 Conclusions

In this paper, we constructed large scale randomly-fluctuating continuum models of ballasted railway tracks. These models allowed to construct dispersion curves for the ballasted layers. The importance of the heterogeneity was stressed very strongly. In particular, it was shown to create relatively slow waves at medium frequencies and even to induce localization at higher frequencies. It was also shown that the heterogeneity of the ballast seems relatively more important that the heterogeneity between ballast and soil. However, this statement should probably be confirmed through a larger set of numerical experiments.

Because of the geometry of the ballast, and because of its heterogeneity, it is absolutely impossible to solve the dynamical equations analytically. Computer simulations are therefore an irreplaceable and unavoidable tool to understand the dynamical behavior of the soil-ballast system. Understanding that behavior, and using more computer simulations.

Acknowledgements and References

The spectral element software used for the simulation in this paper is developed jointly by Centrale-Supélec, CEA Commissariat à l'Energie Atomique and Institut de Physique du Globe de Paris. Within the SINAPS@ project, this development benefits from French state funding managed by the National Research Agency under program RNSR Future Investments bearing reference No. ANR-11-RSNR-0022-04.

References

[1] Connolly DP, Kouroussis G, Laghrouche O, Ho CL, Forde MC. *Benchmarking railway vibrations - Track, vehicle, ground and building effects*, Construction and Building Materials, 2014.

[2] Thi Minh Phuong H, Alart P, Dureisseix D, Saussine G. *A Domain Decomposition method for granular dynamics using discrete elements and application to railway ballast*, Annals of Solid and Structural Mechanics, 87–98, 2011.

[3] Wellmann C, Wriggers P. *A two-scale model of granular materials*, Computer Methods in Applied Mechanics and Engineering, 205-208, 2012.

[4] Sheng X, Jones C, Thompson D. *A theoretical study on the influence of the track on train-induced ground vibration*, Journal of Sound and Vibration, 909–936, 2004.

[5] Freitas da Cunha JP. *Modelling of ballasted railway tracks for high- speed trains*, PhD thesis, Universidad do Minho, 2013.

[6] Bian X, Cheng C, Jiang J, Chen R, Chen Y. *Numerical analysis of soil vibrations due to trains moving at critical speed*, Acta Geotechnica, 281– 294, 2015.

[7] Arlaud E, D'Aguiar S, Balmes E. *Validation of a reduced model of railway track allowing long 3D dynamic calculation of train-track interaction*, Computer Methods and Recent Advances in Geomechanics, 1193–1198, 2014.

[8] Anbazhagan P., Indraratna Buddhima N., Amarajeevi G. *Characterization of clean and fouled rail track ballast subsurface using seismic surface survey method: Model and field studies*, Faculty of Engineering-Papers (Archive), 1-11, 2011.

[9] Jia X, Caroli C, Velicky B. *Ultrasound Propagation in Externally Stressed Granular Media*, Physical Review Letters, 1863–1866, 1999.

[10] Lawney B. P.,Luding S. *Frequency filtering in disordered granular chains*, Acta Mech, 2385–2407, 2014.

[11] Shin H, Santamarina JC. *The Role of Particle Angularity on the Mechanical Behavior of Granular Mixtures*. J Geotech Geoenvironmental Eng. 2012;139(2):483.

[12] Goddard JD. *Nonlinear Elasticity and Pressure-Dependent Wave Speeds in Granular Media*, Proceedings of the Royal Society A: Mathematical, Physical and Engineering Sciences, 105–131, 1990.

[13] Leibig M. *Model for the propagation of sound in granular materials*, Physical Review, 1647–1656, 1994.

[14] Aki K. *Analysis of the seismic coda of local earthquakes as scattered waves*, Journal of Geohysical Research, 615–631, 1969.

[15] de Abreu Corrêa L, Cottereau R, Quezada JC, Costa d'Aguiar S, Voivret C. *Randomly-fluctuating heterogeneous continuum model of a granular medium*, Computational Mechanics Submitted for publication.

[16] Komatitsch, D., Ritsema, J.,Tromp, J.. *The spectral-element method, Beowulf computing, and global seismology*, Science, 1737-1742, 2002.

[17] Komatitsch D. *The spectral-element method in seismology*, Geophysical Monograph Series, 205–227, 2005.

[18] Jacob X, Aleshin V, Tournat V, Leclaire P, Lauriks W, Gusev VE. *Acoustic probing of the jamming transition in an unconsolidated granular medium*, Phys Rev Lett, 1–4, 2008.