

Generalized Inverse Impulse Response for dynamic monitoring and control of structures

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Résumé — Real-time monitoring and control of solids and structures, based on the the solution of the equations of motion, remains a challenge mainly due to its computational complexity. For such a purpose, both direct and inverse formulations of the dynamic problem must be addressed. In this work, we propose computing parametric solutions of the inverse problem by combining classic regularization techniques and the Proper Generalized Decomposition method. This approach could potentially allow for the design of a *generalized* control system, which might be able to control the structure for changing conditions, defined by the parameters.

Mots clés — Structural dynamics, inverse problems, control, monitoring, reduced order modeling, real-time computing, parametric solutions, PGD.

1 Introduction

The computational complexity associated to the solution of the equations of motion has been a concern for several decades. This has led to wide variety of methods, not always well-suited for real-time computing. Therefore, new approaches are required in order to allow for real-time monitoring and control of solids and structures. Numerous potential applications could be then envisaged, ranging from soft robotics to hybrid laboratories. Hybrid laboratories are a combination of experimental laboratories coupled with real time simulations. This approach can reduce the cost of the experiments by simulating the behaviour of a part of the structure instead of realizing it. Therefore, the simulation is done for the part of the system where the behaviour is known and suitable to be computed and the experiment is performed in the part of the system where complex or not well known physics may happen[1][8]. The coupling between the simulation and the experiment is made by an actuator, which transmits the communication between the simulation device and the experiment, transforming forces or displacements coming from the experiment to data and vice versa, data to forces or displacements.

Real time control and monitoring of forces and displacements implies a fast, robust and reliable solution procedure. The simulation requires the solution of the structural dynamic problem in a time interval shorter than the actuator time, which is in the order of a kHz. On the other hand, parametric solutions are an interesting tool that could reduce the operation time in applications where the system parameters may change with certain frequency. Classical methods, although they are well-established and may be very useful in many applications, may not be well-suited for the real-time context. Time-integration schemes may not reach the execution times required. Modal methods might be a good solution for real time applications, but both integration and modal methods fail in computing parametric solutions when more than a few number of parameters are considered.

Pre-computing the solution for all the parameters values in a certain interval is clearly an advantage when fast solutions are sought. Model order reduction methods as Proper Orthogonal Decomposition (POD) [3] or Reduced Model Basis (RBM) [10] are some of the methods applied in parametric problems that produce good results. Proper Generalized Decomposition (PGD) [5], which is also a model order reduction method, brings some advantages to solve parametric models. The transfer function is widely used in structural dynamics to compute the displacements when an external force is applied. This approach can transform linear differential equations in algebraic operations. For real time applications applied in time domain, this function is called impulse response, with which we can compute the

displacements without the benefits of the frequency domain, but avoiding transforming the data to that domain. In multidimensional problems, one can talk about generalized impulse response [2]. The term *generalized* comprehends the dependence of the impulse response on some pre-selected parameters. In dynamical problems, parameters as Young's modulus, damping factor or boundary conditions may be of interest in some analysis.

The objective of this work is to obtain the generalized inverse impulse response. On the contrary of the impulse response, the inverse impulse response links known displacements with unknown external applied forces, therefore an inverse problem must be solved. In this work, the PGD is applied to deal with the parametric nature of the problem in combination with Tikhonov's regularization, which is used to treat the inverse problem. Both direct and inverse generalized impulse responses are a powerful tool for real time control and monitoring of structures, in particular in the context of hybrid simulation.

The rest of the paper is organized as follows : section 2 deals with the direct generalized impulse response, gives an overview about the existing methods and shows how to applied PGD to compute the generalized impulse response. Section 3 address the inverse problem that arises in the computation of the generalized inverse impulse response. Finally, some results and conclusions are shown in the last two sections.

2 The generalized impulse response

In this section the procedure applied to compute the generalized impulse response of a structure is recalled. First, the dynamic equation in time domain is transformed to its equivalent in frequency domain ; this approach is more suitable for the solution of the parametric problem. Then, the transfer function is sought in a parametric form by using the model order reduction technique PGD. Finally, the generalized impulse response is obtained by transforming the transfer function to the time domain by applying Fourier's theory.

The classical methods that can be used to compute the impulse response could be classified depending on how are they approaching the problem : some deal with it in time domain as Newmark's β method [16], some in the frequency domain as the harmonic analysis [11] or they can also apply some modal transformation.

Time integration schemes give a good solution without the need of any transformation of the dynamic equation, but, in real time applications, they are computationally expensive. Modal superposition method is one of the most used methods in dynamic analysis. This method takes profit of eigenvectors properties to transform the system in some *new coordinates* where equations are uncoupled and the resolution time can be reduced. Among the disadvantages of the modal method can be found the case when not proportional damping is considered and a quadratic problem must be solved [13]. In addition, changing conditions (i.e. changing parameters) cannot be managed in real-time due to the cost associated to extracting and projecting the equations onto the new eigenbasis. When more than a few parameters are considered, the so-called *curse of dimensionality* makes unaffordable the solution by classical methods.

Model reduction techniques appear to deal with the dimensionality issue, by lowering the computational cost. Among the model reduction methods we can cite the Proper Orthogonal Decomposition (POD) or the Reduced Basis Method (RBM), which are widely used. Previously cited modal method can be also seen as a model reduction method when the number of its terms are truncated. The general procedure of *a posteriori* model reduction methods is to generate certain solutions to the problem and then find a suitable reduced basis. The parametric domain must somehow explored in order to guarantee the pertinency of the reduced basis for any possible parameter combination. PGD method, on the contrary, is an *a priori* method, that is, its algorithm is able to extract the parametric solution in a separated tensor format without the need of sampling the parametric domain [6]. The strategy followed by PGD algorithm is an iterative enrichment procedure which avoids computing the solution one by one for all the parameters possibilities, allowing then to deal with parametric solutions of several parameters.

2.1 Computing the generalized transfer function

PGD algorithm computes the generalized solution by an iterative procedure, which is more suitable to be applied in frequency domain than in time domain. Therefore, the harmonic method is used to set the

problem in the frequency domain, where the PGD method will be applied to obtain the transfer function in a parametric form. In the harmonic method, one assumes that the excitation can be expressed as a sum of n harmonics, so the resultant system can be written as :

$$\mathbf{f}(t) = \mathbf{f}_s p(t).$$

Applying Fourier transform on the equation of motion, n uncoupled equations (i.e. one for each frequency) are obtained in the frequency domain :

$$(-\omega^2 \mathbf{M} + i\omega \mathbf{C} + \mathbf{K}) \mathbf{h} = \mathbf{f}_s, \quad (1)$$

where one can recognize the inertial, damping, elastic and external applied forces terms, respectively. In addition, \mathbf{h} is known as the transfer function and \mathbf{f}_s collects the spacial coefficients of \mathbf{f} . PGD algorithm is applied then in the equation (1) to find the parametric solution, $\mathbf{h}(\omega)$, in a frequency band $\omega \in [\omega^-, \omega^+]$. The method builds the solution as a separate representation, it is, as a sum of multiplied functions, where each function depends on one parameter, e.g. the Young modulus in the present case. The separated form of the solution reads :

$$\mathbf{h}(\omega, E) = \sum_{i=1}^n \mathbf{X}_i W_i(\omega) \mathcal{E}_i(E), \quad (2)$$

where \mathbf{X} is a vector that collects the nodal generalized space functions, W is a function which depends on the frequency and \mathcal{E} is a function which depends on Young's modulus.

2.2 Computing the generalized impulse response. On line application

Once the generalized transfer function is computed, the impulse response can be easily obtained by applying Fourier theory. For the sake of notation simplicity, we use $\mathbf{h}(\omega)$ and $\mathbf{h}(t)$ being one the Fourier transform of the other, and the dependence on t and ω is explicitly written :

$$\mathbf{h}(\omega) = \int_{-\infty}^{\infty} \mathbf{h}(t) e^{-i\omega t} dt \quad ; \quad \mathbf{h}(t) = \frac{1}{2\pi} \int_{-\infty}^{\infty} \mathbf{h}(\omega) e^{i\omega t} d\omega.$$

Applying Fourier relations to Eq. (2) it reads :

$$\mathbf{h}(t, E) = \mathcal{F}^{-1} \left(\sum_{i=1}^n \mathbf{X}_i W_i(\omega) \mathcal{E}_i(E) \right) = \sum_{i=1}^n \mathbf{X}_i \mathcal{F}^{-1} (W_i(\omega)) \mathcal{E}_i(E) = \sum_{i=1}^n \mathbf{X}_i W_i(t) \mathcal{E}_i(E),$$

where \mathcal{F}^{-1} represents the inverse Fourier transform. Finally, to compute displacements in real time, the generalized impulse response must be recovered and Duhamel's theory applied, resulting the convolution :

$$\mathbf{u}(t, E) = \int_0^t p(t - \tau) \mathbf{h}(\tau, E) d\tau. \quad (3)$$

Note that, if one only needs the displacements on a single degree of freedom, u_j , the correspondent component of \mathbf{h} must be selected :

$$u_j(t, E) = \int_0^t p(t - \tau) h_j(\tau, E) d\tau.$$

3 Computing the generalized inverse impulse response

In the previous section the function which links known forces with unknown displacements has been computed. In this section, the procedure to compute the function which links known displacements with unknown forces will be developed. This is an important function that, in control applications, allows to compute the required force from displacement measures. This function, called inverse impulse response is obtained from (4), which when considering a single displacement coordinate and a single force coordinate writes :

$$p(t) = \int_0^t u(t - \tau) g(\tau) d\tau, \quad (4)$$

which results in solving an inverse problem [14], where several works have been carried out to solve it [4] [7] [9] [15]. Among all the existing techniques, one of the most used is the Tikhonov's regularization [12]. From a set of measures of forces and displacements, the method finds the best solution by least squares technique minimizing the functional Π :

$$\Pi(g) = \sum_{i=1}^m (u_i(t) * g(t) - p_i(t))^2 + \lambda S(g).$$

where $S(g)$ is some imposed condition over $g(t)$, for example a limit in its norm, m is the number of pairs of forces and displacements and "*" denotes the convolution operation. Minimizing Π respect to $g(t)$, the discrete form reads :

$$\mathbf{g} = (\mathbf{U}^T \mathbf{U} + \lambda \mathbf{D}^T \mathbf{D})^{-1} \mathbf{U}^T \mathbf{p}, \quad (5)$$

where \mathbf{U} is the Toeplitz matrix built from $u_i(t)$, \mathbf{D} comes from the regularization condition imposed over $g(t)$ and \mathbf{p} is the vector coming from $p(t)$.

3.1 Parametric inverse impulse response

Hybrid laboratories couple experiments with simulations, and have a great interest when complex physical behaviours which are difficult to simulate occur in some part of a structure. This complex behaviour is reproduced physically in the laboratory, and the rest of the behaviour of the structure is simulated by coupling both processes by an appropriate device. Therefore, the simulation must be able to compute in real time the corresponding reactive force from the data of the experiment, either forces or displacements. Parametric solutions could be used to accomplish such an objective. In this framework, the coupling condition (e.g. displacements at the interface between the actuator and the physical experiment) might be parametrized, and the reaction force of the rest of the structure computed in terms of such parameters. In addition, analysis and optimization of structures are some of the interests of hybrid laboratories, where different structural configurations may be studied, leading to different parameters. By computing the generalized inverse impulse response one would be able to obtain a certain structural configuration by just evaluating the corresponding parameters. Computing the forces from measured displacements could be done in the *on line* phase by a convolution operation, which is not time consuming.

To solve the minimization problem defined in Eq. (5), a training set of forces and displacements is required, and the component of h that correspond to the measured degree of freedom are selected. Data can be obtained from experiments or from simulations. In this case, since the impulse response $h(t, E)$ can be computed as shown in section 2, training data can be generated by using Eq. (3), which reads in parametric form :

$$u_\ell(t, E) = p_\ell(t) * h(t, E) = p_\ell(t) * \sum_{i=1}^n \alpha_i W_i(t) \mathcal{E}_i(E) = \sum_{i=1}^n \alpha_i [p_\ell(t) * W_i(t)] \mathcal{E}_i(E), \quad (6)$$

for $1 \leq \ell \leq m$, being m the number of training measures and α_i the j -th component of X_i . Observe that the separated structure is preserved in Eq. (6). Next, the functional to be minimized is :

$$\Pi(g) = \sum_{\ell=1}^m \left[\left(\sum_{i=1}^n \alpha_i (p_\ell(t) * W_i(t)) \mathcal{E}_i(E) \right) * g(t, E) - p_\ell(t) \right]^2 + \lambda S^2(g).$$

Thanks to the separated structure of this functional, a separated representation of $g(t, E)$ can be sought with the PGD method :

$$g(t, E) = \sum_{i=1}^n V_i(t) Z_i(E).$$

Finally, once $g(t, E)$ has been obtained, the forces can be computed in real time by applying the parametric version of Eq. (4), that is :

$$p(t) = \int_0^t u(t - \tau) g(\tau, E) d\tau,$$

for any value of the parameter E , by simply evaluating the generalized inverse impulse response.

4 Results

This section shows a brief example in order to illustrate application of the aforementioned method to the structure depicted in Fig. 1. The young modulus and the mass density are taken $E = 10000$ and $\rho = 1$, respectively. A proportional damping is considered. Displacements and rotations are null in nodes 1 and 2. Both impulse responses, from direct and inverse problem respectively, are computed. Then, in

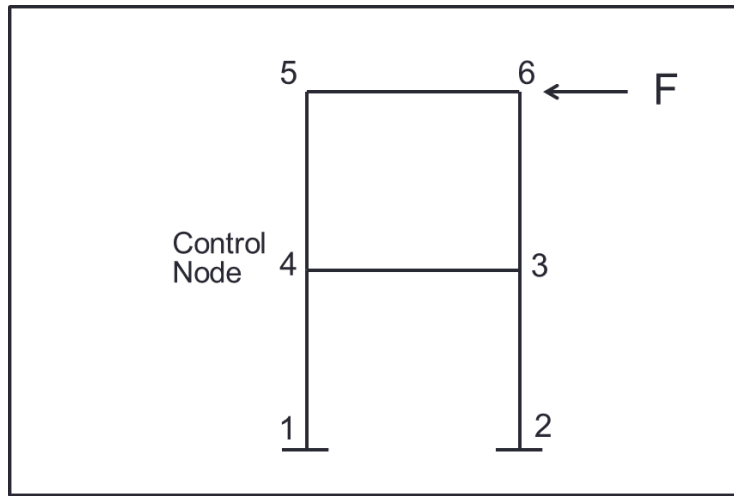


FIGURE 1 – Schematics of a beam structure considered in the example.

an *online* phase, a force \mathbf{F} is applied on x direction on node 6, and displacements u_{test} are computed. To verify the procedure, the original force \mathbf{F} is recovered by convolving u_{test} with $g(t)$. Note that both u_{test} and \mathbf{F} are computed in real time. Results are shown in the Fig. 2.

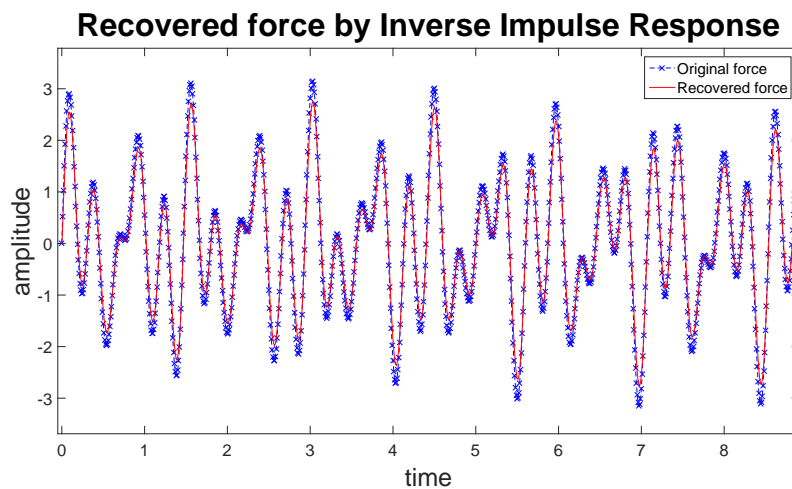


FIGURE 2 – Recovering original force

5 Conclusions

Parametric solutions have proved its strong points when dealing with real time problems. Pre-computing the whole solution of a problem is clearly an advantage when the system parameters can change with certain frequency. Recent developed model reduction method PGD allows to build a parametric solution avoiding the curse of dimensionality. Impulse response is the function that links unit impulse forces with caused displacements. By a simple convolution operation, it gives the response in displacements for a known external applied force. Its computation is a direct problem, where several methods may be used to calculate it. On the other hand, the impulse response that links known displacements with external forces comes from the solution of an inverse problem, and regularization techniques have been applied to solve

it. By computing generalized impulse responses, both direct and inverse, which its computation rests in an *offline* phase, we are able to compute in real time forces and displacements in structures under elastic behavior.

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