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TOPOLEV: Topological optimization using level-set method

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Summary — Weight reduction is a very important issue in the industry, particularly in the industry of transport vehicles. To meet this objective, ESI has developed a disruptive and innovative optimization tool based on the technology of the level-set [[5], [6]]. Unlike existing methods (homogenization, and flavors: power law, SIMP etc. [[1], [8]]), the level-set representation allows an accurate sharp knowledge of the boundary location, thus we are able to have a large scope of geometrical constraints of the shape, or to include manufacturing constraints involving precise knowledge of the shape like casting or additive manufacturing processes.
Keywords — Shape optimization, Topological optimization, Level-Set, TOPOLEV, casting, additive manufacturing.

1. Introduction

Weight reduction is a major issue for the industry. Especially in Vehicles industry where the reduction of energy consumption is engaged to reduce environmental impact. The first goal of the topological optimization is to find new shapes that minimize an objective (most of the time the volume or the mass) but preserve a sufficient level of performance.

2. Level-set representation

2.1. Shape representation

Most of existing optimization software solutions use a density approach to represent the shape on a fixed mesh. In our method we have chosen to use the now well-known level-set method for our shape representation.

Let be an open bounded $D \subset \mathbb{R}^3$ we called 'design space', is the maximum space where to search for an optimal shape. Let be Ω an admissible shape then $\Omega \subset D$.

In order to represent the shape Ω in the design space, we defined a function ψ (the level-set function) on D such as:

$$\begin{cases} \psi(x) = 0 \Leftrightarrow x \in \partial\Omega \\ \psi(x) < 0 \Leftrightarrow x \in \Omega \\ \psi(x) > 0 \Leftrightarrow x \in (D \backslash \bar{\Omega}) \end{cases} \tag{1}$$

A good common choice for a function that fits these rules is to use the signed distance to the boundary. This kind of choice for level-set function implies a strong regularity of the function, but also helps for geometrical criteria.

The evolution of the shape is governed by the now well-known Hamilton Jacobi problem:

$$\frac{\partial\psi}{\partial t} - v|\nabla\psi| = 0 \tag{2}$$

In this equation, t is a fictitious time representing the optimization step increment. v is called 'descent direction' and is given by the optimizer.

2.2. Computation of the descent direction

The main goal of the optimizer is to compute the descent direction and has been studied a lot ([4], [9]). With the standard methods, a lot of algorithms are available (SLP, MMA, SQP, MFD, Uzawa, etc.) but all these methods rely on explicit optimization parameters. With the use of level-set, the optimization parameter is implicit (the place where the level-set function is null).

The optimization problem is written as:

$$\begin{cases} \min\limits_{\Omega} f(\Omega) \\ g_i(\Omega) \leq 0 \ \forall i \end{cases} \tag{3}$$

Where f is called the objective function and g_i are the constraints, the union of objective and constraints are called 'criteria' of the optimization.

In order to compute the descent direction we need to evaluate all the criteria, and to solve the problem (3), most of algorithm also need the gradient. By generalizing the gradient formulation given by [1] we obtain the expression of a linear static mechanical criterion as:

$$J(\Omega) = \left(\int_{\Omega} j dx\right)^{\beta} + \left(\int_{\partial\Omega} l dx\right)^{\gamma} \tag{4}$$

And the formulation of its gradient:

$$\begin{aligned} J'(\Omega)(\theta) = &\int_{\partial\Omega_N} \theta.n\left(\beta C_{j0} j + \sigma_p : \varepsilon_u - p.f - \frac{\partial(p.g)}{\partial n} - \kappa(p.g) + \gamma C_{l0}\left(\frac{\partial l}{\partial n} + \kappa l\right)\right) \\ &+ \int_{\partial\Omega_D} \theta.n\left(\beta C_{j0} j + \sigma_p : \varepsilon_u - p.f + \gamma C_{l0}\left(\frac{\partial l}{\partial n} + \kappa l\right) - \frac{\partial(h\sigma_p)}{\partial n} n - \kappa(h\sigma_p) n\right) \end{aligned} \tag{5}$$

In this equation, $C_{j0} = \left(\int_{\Omega} j \, dx\right)^{\beta-1}$ and $C_{l0} = \left(\int_{\partial\Omega} l \, dx\right)^{\gamma-1}$.

The mean curvature κ and the normal n to the boundary are computed using geometrical properties of the level-set.

u is computed as the solution of the static linear problem:

$$\begin{cases} -\nabla.\sigma_u = f & in \ \Omega \\ u = h & on \ \partial\Omega_D \\ \sigma_u.n = g & on \ \partial\Omega_N \end{cases} \tag{6}$$

And p is the solution of the adjoint problem given by:

$$\begin{cases} -\nabla.\sigma_p = -\beta C_{j0} j'(u) & in\ \Omega \\ p = 0 & on\ \partial\Omega_D \\ \sigma_p.n = -\gamma C_{l0} l'(u) & on\ \partial\Omega_N \end{cases} \quad \textbf{(7)}$$

The main advantage of this adjoint problem is that the left-hand side of the problem is the same as the direct mechanical problem. Thus the matrix factorization done for the direct problem could be used for the computation of the adjoint problem.

3. Some manufacturing constraints

3.1. Maximum thickness

According to [3], the maximum thickness T_{max} could be interpreted such that there is no point inside the shape which are the center of a ball of diameter T_{max} fully covered by material. In other terms, there is no point inside the shape that is farther than $T_{max}/2$ from a boundary.

As the level-set function is the signed distance to the boundary, with a negative sign inside the shape, we could express the criterion as:

$$-2\psi(x) \le T_{max}, \forall x \in \Omega \quad \textbf{(8)}$$

This is a semi-infinite criterion, therefore the integral form of the criterion use is:

$$J_{MT}(\Omega) = \left(\frac{\int_\Omega f(\psi)\big(-2\psi(x)\big)^\alpha}{\int_\Omega f(\psi)} \right)^{\frac{1}{\alpha}} \le T_{max} \quad \textbf{(9)}$$

With $\alpha > 1$

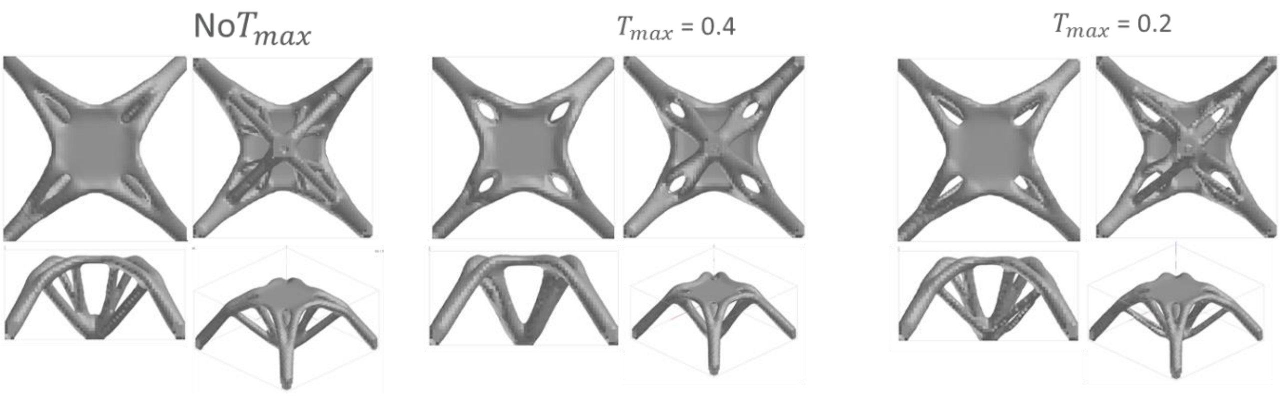

Figure 1 - Impact of maximum thickness threshold on design

Where the function $f(\psi)$ is use to 'activate' the terms only at the location where the constraint is violated. In order to have a function with sufficient regularity we use:

$$f(\psi) = \frac{1}{2}\left(1 - \tanh\left(\frac{-2\psi(x) - T_{max}}{\beta T_{max}}\right)\right) \quad \textbf{(10)}$$

The parameter β is used to control the regularization of the function; if $\beta \to +\infty$ the function tend to the Heaviside function.

Some example could be seen on figure 2, which shows the impact of the threshold on the topology of the solution.

3.2. Angular control

An angular criterion control could be interpreted as for a given direction d and a given maximum angle ϕ_{max} the angle that forms the boundary surface with the normal plan to the direction should not exceed the maximum angle.

In other terms the criterion could be expressed as:

$$d.n \leq \sin(\phi_{max}), \forall x \in \partial\Omega \tag{11}$$

For example, for casting consideration, we will use a positive angle (typically 3°), but if we use additive manufacturing constraint we will use a negative angle (for example -35° for powder bed)

As the maximum thickness constraint, this constraint must be expressed as a semi-infinite constraint with an integral form as:

$$J_{AC}(\Omega) = \left(\frac{\int_{\partial\Omega} f(d.n)(d.n)^{\alpha}}{\int_{\partial\Omega} f(d.n)} \right)^{\frac{1}{\alpha}} \leq \sin(\phi_{max}) \tag{12}$$

With the function $f(\psi)$:

$$f(d.n) = \frac{1}{2}\left(1 - \tanh\left(\frac{d.n - \sin(\phi_{max})}{\beta \sin(\phi_{max})}\right)\right) \tag{13}$$

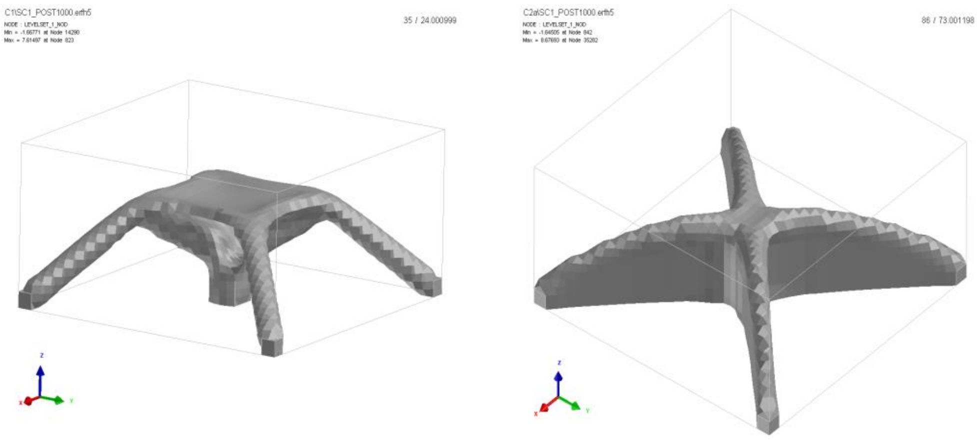

Figure 2 - (left) no angular control, (right) direction Z, angle of 0

On Figure 2, we could see the result of two optimizations with and without the angular control. With the angle set to 0, we allow in this case to have vertical wall but no part of the shape boundary could face the bottom.

4. Industrial result

4.1. Automobile test case

In this case, only the interior part of the bracket is optimized (on Figure 3, the white part)

The objective is to minimize the volume with constraints on stiffness (control the displacement of loaded nodes), constraints of vibration (control the first Eigen Frequency), angular constraint (in Z direction, angle 0°), maximum thickness of the components, and control of the maximum Von Mises.

Figure 3 shows the initial shape and the optimized shape remeshed. The mass gain is about 25%

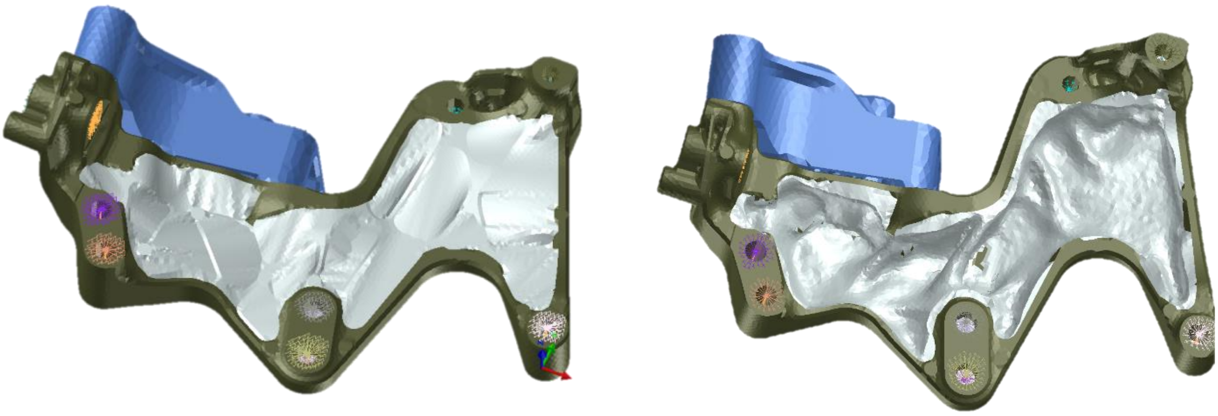

Figure 3 – Initial geometry (left) Optimized shape remeshed (right)

4.2. Aeronautic test case

This case is an engine bracket which has 14 mechanical load cases.

In this test case, the objective is to minimize the sum over the 14 load cases of the compliance (i.e. maximization of the rigidity) the constraints are to constraint the displacement of certain nodes in each load case and constraint the mass in order to have reduction of 30%.

Only small parts of this case are set not optimizable, (in yellow on the figure)

Figure 4 shows the shape at initial and final stages.

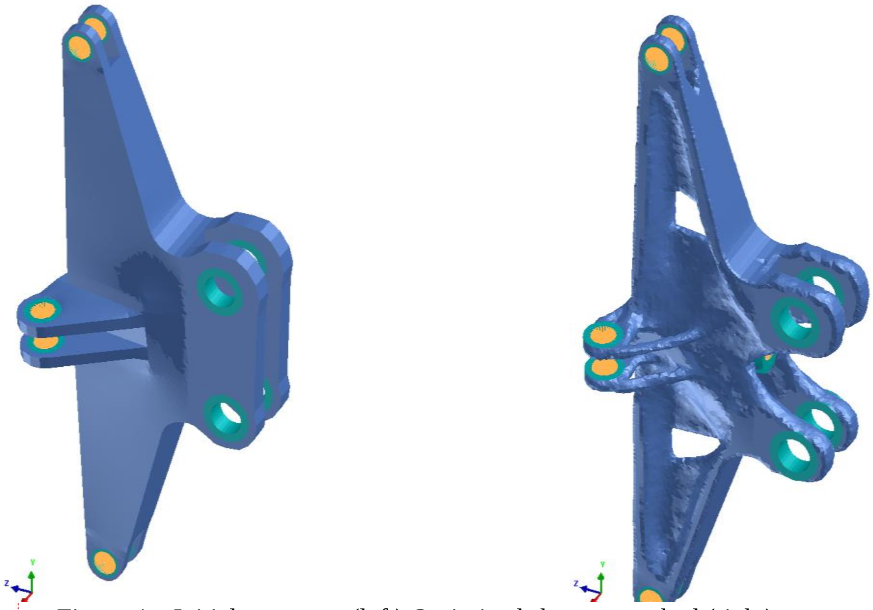

Figure 4 – Initial geometry (left) Optimized shape remeshed (right)

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