

A lattice-based approach for hydraulic fracturing simulation in jointed materials

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Abstract — Characterizing the path of a hydraulic fracture in a heterogeneous/jointed medium is one of the challenges of current research on hydraulic fracturing. We present here a 2D lattice hydro-mechanical model for this purpose. Natural joints are represented by introducing elements with a new plastic-damage behaviour. The action of the fluid pressure on the skeleton is represented by using Biot's theory. The interactions of cracks on the fluid flow are represented by considering a Poiseuille's flow between two parallel plates. Numerical coupling is achieved with a staggered coupling scheme.

Keywords — Lattice analysis, Fracture, Fluid-structure interaction, Jointed material.

1 Introduction

Crack propagation under fluid injection is a coupled and complex problem with various applications, from magma transport in the lithosphere to oil and gas reservoir stimulation from the 40's [5]. Different coupled effects have been studied in the literature such as the interaction between different stimulated cracks [17], the influence of the spatial variation of the rock mechanical properties on the crack extensions [16], the proppant capacity to fill the crack space and its influence on the crack openings [4] or the influence of dynamic sollicitations on the variation of the rock global permeability [3].

For homogeneous materials, different analytical solutions have been proposed for bi-wing crack configurations. For the KGD^1 configuration, see e.g. [15] or [8]. For the PKN^2 configuration, see e.g. [20], [19] or [1]. For a comparison of the two, see e.g. [9]. These analytical solutions predict the width and extend of hydraulically-induced fractures by taking into account the fluid transfer within the matrix through a Carter's *leak-off* coefficient [14] but to solve the problem analytically, different asymptotic regimes are distinguished. For instance the fluid flow may be dominated by the leak-off or the fluid may preferentially be stored within the propagating crack. The mechanical energy may be preferentially dissipated through the matrix fracture (toughness-dominated) or through frictional shear forces within the fluid (viscosity-dominated) (e.g. see [2], for a study of a toughness-dominated hydraulic fracture with leak-off). For intermediate cases, the global model cannot be solved analytically and numerical modeling is needed to characterise width and extend of hydraulically-induced fractures and there interactions with the natural network of pre-existing joints.

The mechanical and hydraulic behaviour of a rock formation is highly dominated by the natural network of pre-existing joints. This is particularly the case for source rocks, which have been intensively fractured hydraulically for oil and gas extraction (see e.g. [6], for a description of middle and upper devonian gas shales of the Appalachian basin, [7], for a description of natural fractures in a Barnett shale or [21], for the influence of geologic discontinuities on hydraulic fracture propagation). Natural joints may have been cemented by geological fluid flows and the global permeability of the system will highly depend on the capacity of the hydraulic fracture to reactive these natural joints.

This research study aims at developing a lattice-type numerical model allowing the simulation of crack propagation under fluid injection in a quasi-brittle heterogeneous medium. A lattice-based mo-

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deling description has been chosen because it has been shown in previous studies that this mesoscale approach is capable not only to provide consistent global responses (e.g. Force v.s. CMOD responses, see [11], or [13]) but also to capture the local failure process realistically (see [13], or [18]). Moreover, this numerical model is based on a dual Voronoï/Delaunay description, which is very efficient to represent dual mechanical/hydraulic couplings [10]. Therefore, this numerical tool will be used here to get a better understanding of initiation and propagation conditions of cracks in rock materials presenting natural joints where the coupling between mechanical damage and fluid transfer properties are at stake.

This paper is organized as follows. After having briefly recalled in Section 2 the lattice model used in this paper, we proceed in Section 3 to the comparisons to analytical solutions. It is found that the model is consistent with LEFM in the pure mechanical case, and with analytical solutions from the literature in the case where the leak off is dominant. Section 4 presents the influence of an inclined natural joint of finite length crossed by the fracture. It is shown that the crack follows the joint and that propagation starts again from the tip.

2 Lattice modeling

2.1 Mechanical description

2.1.1 Matrix description

A 2D plane-stress lattice model is used to characterize the initiation and propagation conditions of cracks in rock materials presenting natural joints. This lattice model is based on the numerical framework proposed by Grassl and Jirasek [12]. It has been shown in previous studies that this mesoscale approach is capable not only to provide consistent global responses (e.g. Force v.s. CMOD responses) [11, 13] but also to capture the local failure process realistically [13] for quasi-brittle materials such as concrete or rocks. The numerical procedure is briefly presented in this section. The reader may refer to references [12, 11, 13, 18] for further details.

The matrix is supposed to be homogeneous at the scale of the study and the lattice is made of beam elements, which idealize the material structure. The matrix structure is meshed by randomly locating nodes in the domain, such that a minimum distance is enforced. The lattice elements result then from a Delaunay triangulation (solid lines in figure 1a) whereby the middle cross-sections of the lattice elements are the edges of the polygons of the dual Voronoi tessellation (dashed lines in figure 1a).

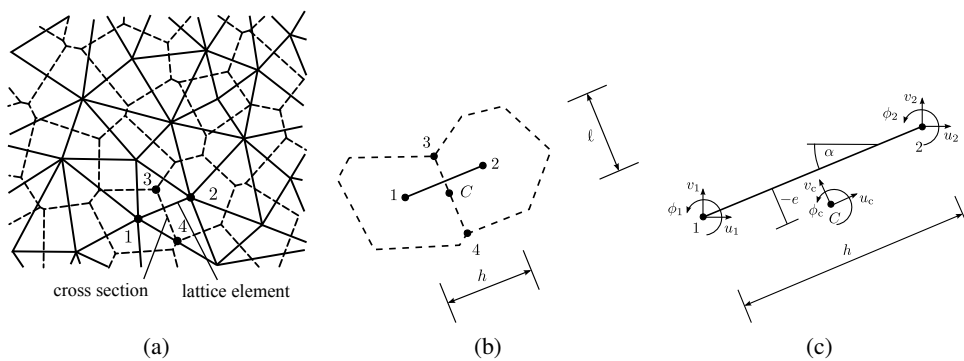


FIGURE 1 – (1a) Set of lattice elements (solid lines) with middle cross-sections (dashed lines) obtained from the Voronoi tessellation of the domain. (1b) and (1c) Lattice element in the global coordinate system (Reproduced from [11]).

Each node has three degrees of freedom : two translations (u, v) and one rotation (ϕ) as depicted in figure 1c. In the global coordinate system, the degrees of freedom of nodes 1 and 2, noted $u_e = (u_1, v_1, \phi_1, u_2, v_2, \phi_2)^T$, are linked to the displacement jumps in the local coordinate system of point C, $u_c = (u_c, v_c, \phi_c)^T$. See [12, 11, 13, 18] for details.

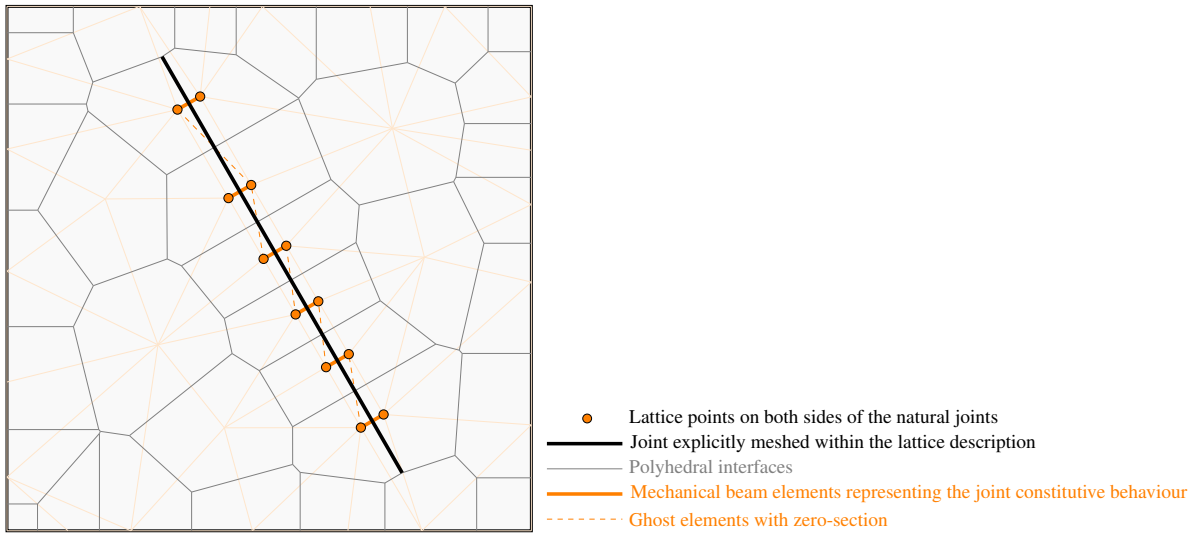


FIGURE 2 – Lattice description of a 30° inclined natural joint of finite length.

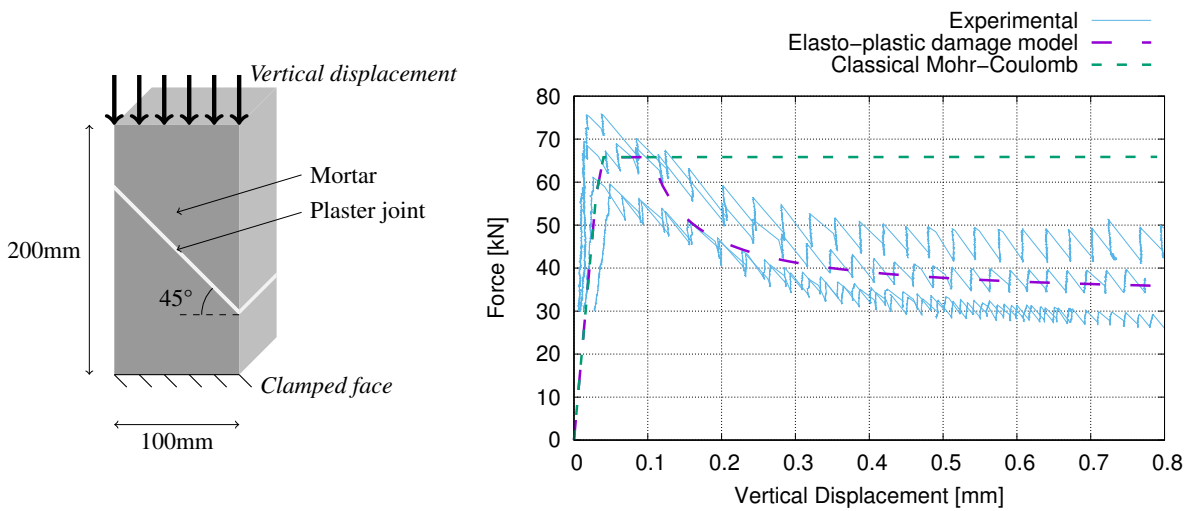


FIGURE 3 – Comparison between experimental and numerical results for an indirect shear test.

An isotropic damage model is used to describe the mechanical response of lattice element within the matrix.

The elastic constants and the model parameters in the damage models are calibrated from an inverse analysis technique.

2.2 Natural joint description

Natural joints are explicitly described within the model with beam elements presenting a new elasto-plastic damage constitutive law (figure 2). The originality of the model lies in the coupling between mechanical damage under normal strain and plasticity under tangential strain. Mechanical damage induces a decrease of the material cohesion whereas the plastic strains, in both normal and tangential directions, participate to the damage evolution. This new constitutive law is able to reproduce indirect shear experimental tests performed on mortar specimens presenting a plaster joint where a classical Mohr-Coulomb criterion fails (see Figure 3) .

2.3 Hydraulic description

The hydro-mechanical coupling is introduced through a poromechanical framework based on the intrinsic and dual hydro-mechanical description of the lattice model, which is based on a "hydraulic"

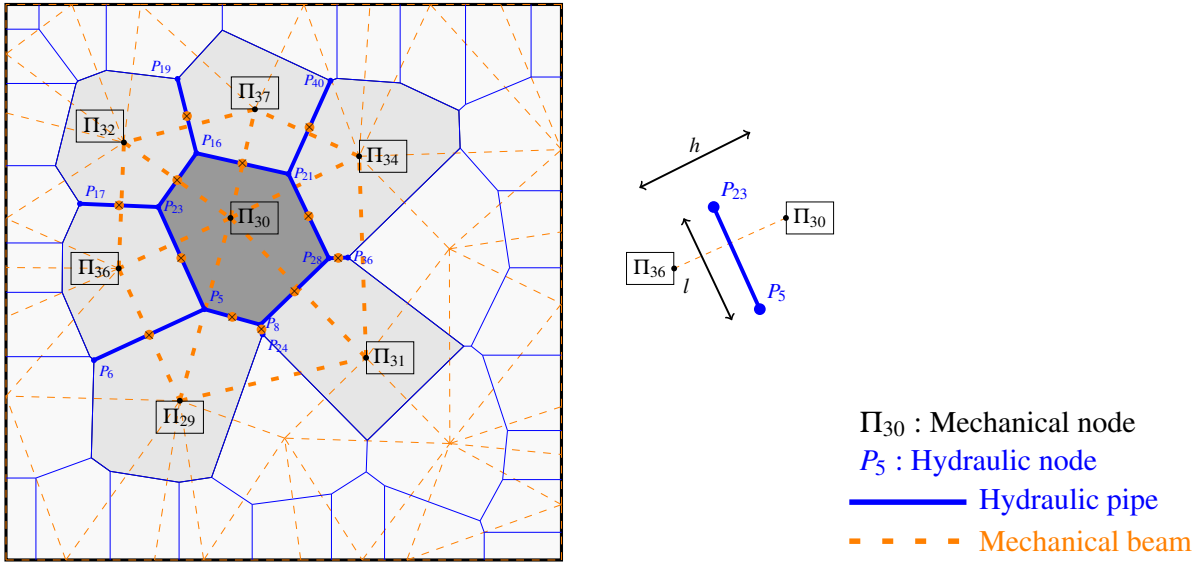


FIGURE 4 – Dual hydro-mechanical lattice description.

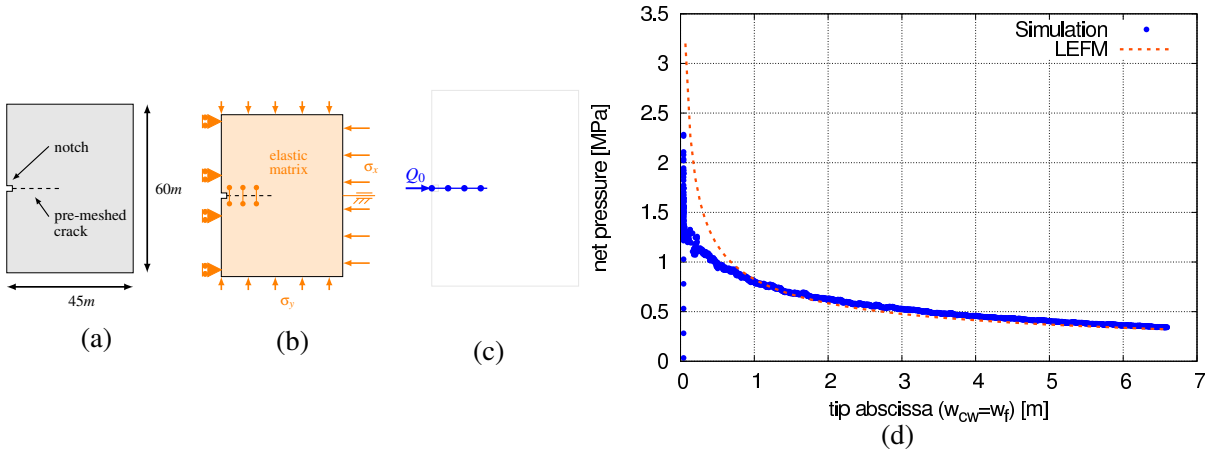


FIGURE 5 – LEFM comparison for an impermeable crack : (a) geometry ; (b)-(c) mechanical and hydraulic boundary conditions ; (d) net pressure vs. crack tip abscissa.

Voronoi tessellation and a "mechanical" Delauney triangulation (figure 4). The total stress links the mechanical stress and the pore pressure through the Biot coefficient of the medium whereas the local permeability, which drives the hydraulic pressure gradient, depends on the local crack openings. The following assumptions are made : the matrix porosity is connected and its volume depends on the fluid pressure ; the fluid saturates the porous medium and is incompressible ; only laminar flows are considered and gravity effects are neglected. The model is simplified by neglecting the effect of deformation in the equation governing fluid flow. Numerical coupling is achieved with a staggered coupling scheme.

3 Comparisons to analytical solutions

3.1 LEFM comparison for an impermeable crack

Linear elastic fracture mechanics (LEFM) links the fluid pressure to the crack extend for quasi-static stable crack propagation. In this section and for LEFM comparison purposes the crack path is pre-meshed within the lattice modeling. The surrounding matrix is assumed perfectly elastic and impermeable. A fluid flow is imposed in a short prenotch and net pressure versus crack tip abscissa are compared in Figure 5. A good agreement is observed as soon as the crack extend is high enough to neglect the fracture process zone influence.

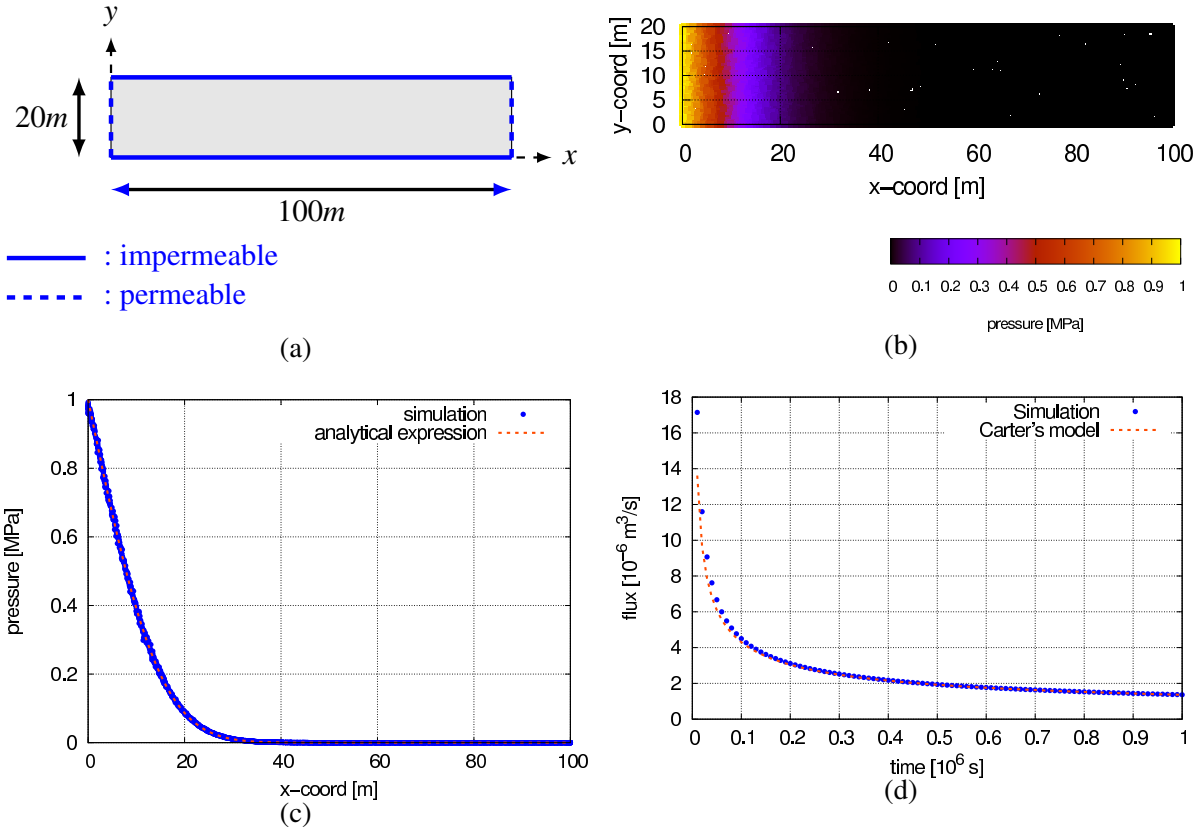


FIGURE 6 – Leak-off representation and comparison with Carter’s model : (a) geometry ; (b) pressure repartition at $t = 10^6s$; (c)-(d) comparison between Carter’s model and the lattice results in term of pressure repartition at $t = 10^6s$ and fluid flux temporal evolution at $x = 0.01m$ position.

3.2 Leak-off representation and comparison with Carter’s model

When an hydraulically stimulated crack propagates within a permeable medium, its extend depends on the so-called *leak-off*, the quantity of fluid which drives out within the matrix. Carter’s model represents the leak-off as an unidimensional diffusive flow perpendicular to the crack lips. For comparison purposes, we propose a simple hydraulic problem where a fully saturated beam is submitted to a pressure gradient (figure 6). A good agreement is observed for this simple diffusive flow problem.

3.3 Analytical comparison for a permeable crack

Bunger *et al.* [2] presents an analytical solution for the study of a toughness-dominated hydraulic fracture with leak-off. The geometry presented in figure 5 is used with a permeable crack. Since the analytical solution presented in [2] is based on brittle fracture, the crack path is still pre-meshed within the lattice modeling. Figure 7 presents the comparison between the analytical solution and the lattice results in term of crack extend evolution with time and crack opening repartition. A good agreement is observed for a toughness-dominated with leak-off regime.

4 Influence of a natural joint on the hydraulic fracture crack path

Since the model has been validated by different analytical comparisons in Section 3, we propose here to study the free propagation of an hydraulically-induced fracture and its interaction with a pre-existing joint. The crack path is not pre-meshed here and is a direct results of the lattice modeling (Figure 8.a) as well as the pressure repartition within the host matrix (Figure 8.b).

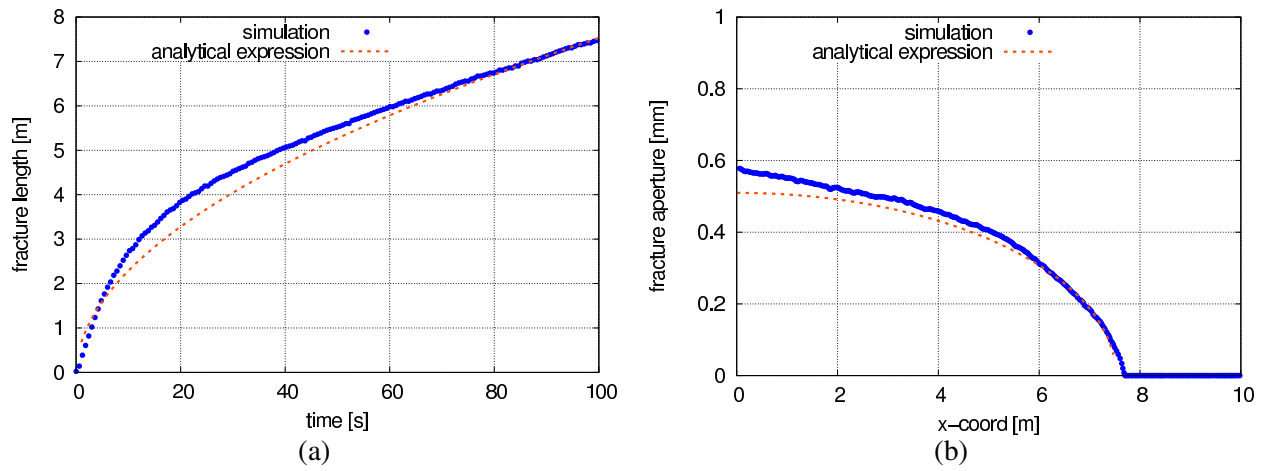


FIGURE 7 – Comparison between the lattice results and an analytical solution for a toughness-dominated hydraulic fracture with leak-off [2] : (a) crack extend evolution with time ; (b) crack opening/aperture repartition at $t = 100s$.

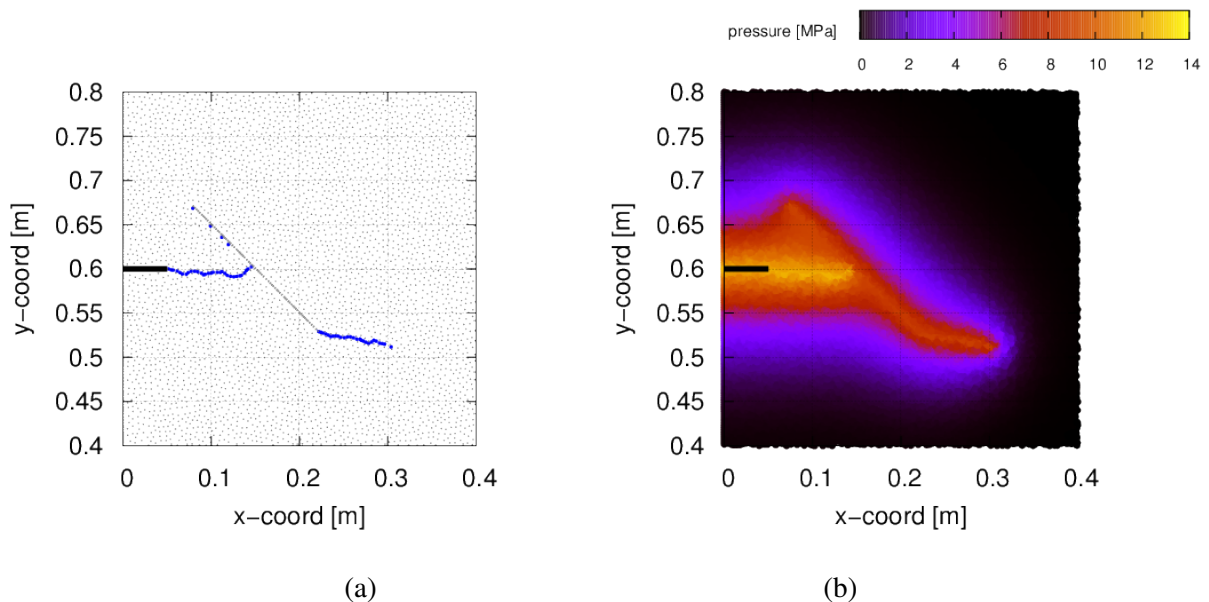


FIGURE 8 – Influence of a 45° inclined joint on the crack path of a hydraulically-induced fracture : (a) crack path after $t = 100s$; (b) pressure repartition after $t = 100s$.

5 Concluding remarks

In this paper, a new hydro-mechanical coupled lattice-based model has been proposed for 2D-simulation of crack propagation induced by fluid injection in heterogeneous quasi-brittle materials. The hydro-mechanical coupling is introduced through a poromechanical framework based on the intrinsic and dual hydro-mechanical description of the lattice model. Different configurations have been studied to test the relevance of the lattice description. It has been shown that it is capable of representing the crack propagation in both impermeable and permeable media by taking into account the leak-off phenomena accurately. A new elasto-plastic damage constitutive law has been proposed to mimic natural joint behaviour. This new constitutive law is able to reproduce indirect shear experimental tests performed on mortar specimens presenting a plaster joint where a classical Mohr-Coulomb criterion fails. Within this framework, the influence of a 45° inclined joint on the crack path of a hydraulically-induced fracture has been studied and pressure repartition within the host matrix has been presented.

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