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Studies on the inverse identification of Voce parameters with indentation test and non-linear shape-manifold learning method

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Résumé — In this present work, the existence of non-uniqe solution to inverse identification of three-parameter Voce law is revealed. Different from dimensional analysis, a pair of "mystical" materials are found through two identification procedures with the aid of non-linear manifold learning algorithm. Considering the non-uniqueness of solution derived with conical indenters when calibrating Hollomon's parameters, spherical indenters are preferably employed in view that one more length scale is involved. The results presented are based on synthetic data without any noise added.
Mots clés — Inverse problem, indentation, manifold-learning, voce law.

1 Introduction

The uniqueness of solution to inverse identification has in recent years become a focal point of interest in indentation literatures, in spite of fruitful results obtained for different materials. After years of study, it turns out to be an indisputable fact that the inverse analysis result is non-unique due to the use of self-similar indenter. Even for a dual (or plural) sharp indenter, the uniqueness assumption is far too less investigated or challenged. This lack of critical verification was attributed to the absence of an explicit relationship between the material properties and the indentation responses.

In Chen's work [4], they proposed an explicit formulation to determine mystical material pairs with distinct elasto-plastic properties while exhibit almost identical indentation responses (mainly focused on P-h curves) even in the use of multi indenters. It has been shown that, even with the established techniques like dual sharp indentation, distinguishing some power-law materials seems to be impossible. They also presented a thin-film indentation analysis as well as an improved spherical indentation technique for specimens with finite thickness and bulk materials, respectively.

The inverse identification of the two-parameter power law (Hollomon's law) has been thoroughly studied in literatures. Recent research work addressed that many important engineering materials deviate significantly from the power law hardening property.

In current study, a Voce law exhibiting a saturation stress at high strain lever is considered. In view of the non-uniqueness of solution derived with conical indenters when calibrating Hollomon's parameters, spherical indenters are preferably employed since one more length scale is involved. In current work, special attention is given to the existence of non-unite inverse solution. The identification procedure is accomplished with the aid of non-linear manifold learning algorithm developed by our research groupe [2, 3], with the help of which, the mystical material pair is found.

2 Overall concept and formulation

2.1 Material model

Without losing generality, an elasto-plastic model with isotropic hardening is assumed in this study. Instead of a power hardening law adopted in [2], in this work, we focus on the Voce relationship which involves one more parameter in the constitutive expression. According to [1], the uniaxial stress-strain

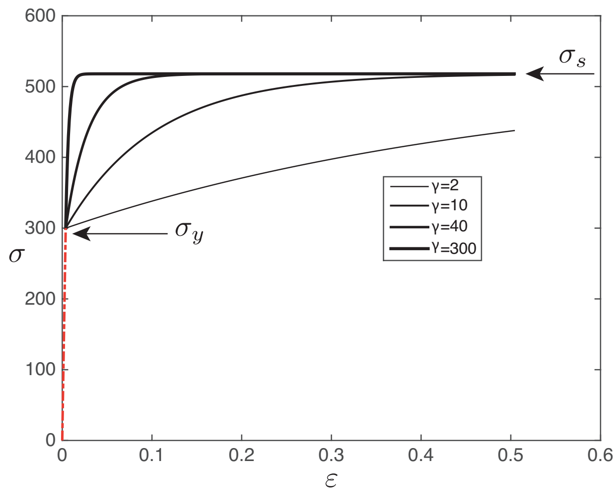


FIGURE 1 – Voce law defined on three parameters

$(\sigma - \varepsilon)$ relationship defined by Voce law reads

$$\begin{cases} \sigma = E\varepsilon \\ \sigma = \frac{\sigma_y}{1-m_1}(1 - m_1 e^{-m_2\varepsilon_p}) \end{cases}, \tag{1}$$

where E is the Young's modulus, ε refers to the total strain, ε_p the plastic stain, σ_y the elastic limit, m_1 and m_2 are two dimensionless material parameters which define the hardening behavior. Another more popular definition which adopted the saturation stress σ_s endows physical meaning to the parameters. The plastic part turns to be

$$\sigma = \sigma_y + (\sigma_s - \sigma_y)(1 - e^{-\gamma\varepsilon_p}). \tag{2}$$

Essentially, the plastic parts defined by (1) and (2) are equivalent, where σ_s, defined as the stress at high strain level (before rupture), is a function with regards to σ_y and m_1, reading $\sigma_s = \frac{1}{1-m_1}\sigma_y$. Like m_2 in (1), the parameter γ controls the rapidity of stress approaching to saturation status. As can be seen in Fig.1, the parameter γ can vary in a rather large range, from 2 to several hundred. This range covers almost all possible hardening materials in engineering practice. For example, the constitutive relation shows a bilinear elasto-plastic property as γ decreases to 2, while an elastic-perfectly plastic property is approximated when γ approaches 300. In the work of Zhang & Wang [1], based on dimensional analysis, (1) is employed to define a dimensionless function independent of m_1 and m_2, by virtue of carefully chosen representative strain ε_r. In current work, for the sake of clear physical meanings, (2) will be employed and large ranges of variations can be assigned to σ_y and σ_s in the choice of different materials.

2.2 Finite element model

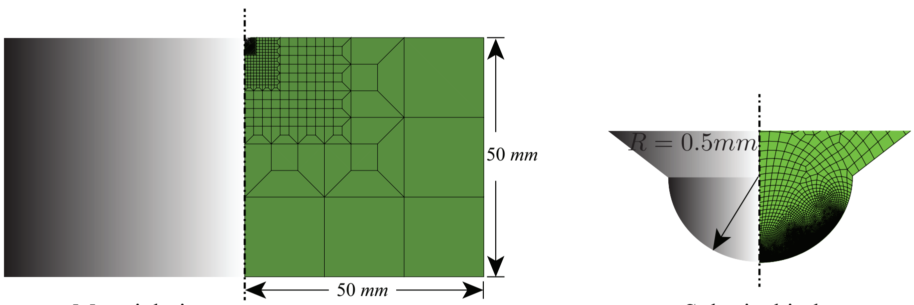


FIGURE 2 – Finite element model of specimen and different indenter tips.

The discretization of indenters and of the specimen are shown in Fig.2. A dense mesh is employed near the interest zone where contact is present between the two pieces. Besides, to approximate the semi-infinite domain condition, a rather large size of the specimen is chosen, $50mm \times 50mm$. The axisymmetric boundary conditions are selected in view of the geometric symmetry of both the specimen and the indenter. The indentation system is modeled with ABAQUS/Standard involving 4394 four-node axisymmetric elements CAX4R for the specimen, 6070 elements for spherical indenter. The contact interface between the two pieces is characterized by Coulomb friction coefficient c which is set to 0.2. For the indenter, Young's modulus $E_i = 600\text{G}Pa$ and Poisson's ratio $\nu_i = 0.23$ are assigned to approximate the elastic properties of Tungsten carbide.

2.3 Identification in Reduced-order space

We apply the method of POD to the collection of imprint shapes obtained by an indentation test. We start with M numerical experiments defined by an appropriate DOE for the varying material parameters. The different imprint shapes are next extracted from the simulation results and each of them is considered as a snapshot vector $\mathbf{s}_i, i = 1, 2 \cdots M$. The snapshot matrix is generated from the centered snapshots $\mathbf{S}$

$$\mathbf{S} = [\mathbf{s}_1 - \bar{\mathbf{s}}, \mathbf{s}_2 - \bar{\mathbf{s}}, \cdots \mathbf{s}_M - \bar{\mathbf{s}}], \tag{3}$$

where $\bar{\mathbf{s}}$ is the mean snapshot $\bar{\mathbf{s}} = \frac{1}{M}\sum_{i=1}^{M} \mathbf{s}_i$. We employ proper orthogonal decomposition (POD) to search the reduced-order space in which evolves the imprint shapes. Singular value decomposition of $\mathbf{S}$ yields

$$\mathbf{S} = \mathbf{\Phi} \mathbf{D} \mathbf{V}^{\mathbf{T}}, \tag{4}$$

where $\mathbf{D}$ contains the singular values d_i ; each column of $\mathbf{\Phi}$ is an eigenvector of the covariance matrix $\mathbf{C} = \mathbf{S}\mathbf{S}^{\mathbf{T}}$, and $\lambda_i = d_i^2$ are the corresponding eigenvalues. The eigenvectors ϕ_i are generally called the POD modes. Each snapshot $\mathbf{s}_i$ can then be accurately reconstructed by a projection basis $\mathbf{\Phi} = [\phi_1, \phi_2 \cdots \phi_M]$

$$\mathbf{s}_i = \bar{\mathbf{s}} + \mathbf{\Phi}\boldsymbol{\alpha}^i = \bar{\mathbf{s}} + \sum_{j=1}^{M} \alpha_j^i \phi_j, \tag{5}$$

where α_j^i is the projection coefficient (or the coordinates in reduced space) for the i^{th} snapshot on the j^{th} mode, given

$$\alpha_j^i = \phi_j^{\mathbf{T}} \mathbf{s}_i, j = 1, 2 \cdots M. \tag{6}$$

In this constructed low-dimensional space, a predictor-corrector manifold walking algorithm is proposed [3] to iteratively locate the local manifold that contains the projection of the experimental result.

3 Numerical example

In current work, to demonstrate the existence of non-unique solution to inverse identification even for ideal situation, synthetic data is employed without adding any noise. For the pseudo-experimental responses, the P-h curve and the imprint are obtained with elastic limit $\sigma_y^{ref} = 300MPa$, saturation stress $\sigma_s = 500MPa$ and isotropic hardening coefficient $\gamma^{ref} = 14$. The FE model presented in Sect.2.2 is employed for simulations, controlling the maximal penetration depth by the end of loading phase $h_{max} = 0.1mm$. Using the local manifold learning "Floating search" algorithm [2, 3], we intend to retrieve back the three "missing" constitutive parameters considering imprint snapshot only. The initial parameter set $A : (\sigma_y, \sigma_s, \gamma) = (430, 780, 100)$ is chosen and the shrinking coefficient is fixed at $\beta = 0.8$.

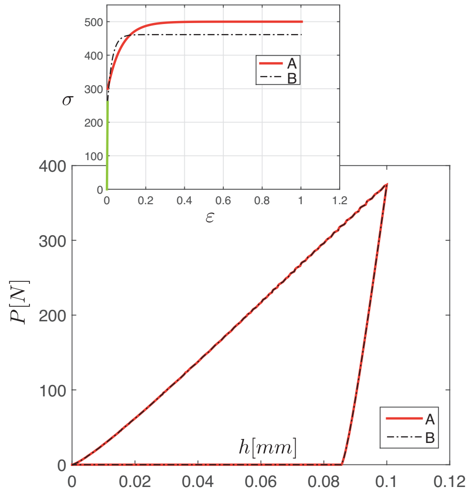
Besides, another combination of material parameters $B : (\sigma_y, \sigma_s, \gamma) = (200, 800, 140)$ is also chosen for the initial value for the purpose of ruling out possible influence of the choice of initial point in inverse optimization. The corresponding identification result is provided, in comparison with the one obtained with staring point A, in Tab.1, showing totally different identification results.

As observed, all the three parameters are almost exactly retrieved from the staring point A, showing a maximal error of less than 1%, while a big deviation from the reference value is observed for γ, about 200%. Two possible reasons should explain this phenomenon : either the iteration does not converge, or, there exists more than one solution that minimize the discrepancy between experimental and simulated indentation responses. In this regard, both the indentation curve and the final residual imprint are compared in Fig.3. We point out that the two responses are almost overlapping with each other while the constitutive behavior are well distinguished, referring to top-left corner in Fig.3.

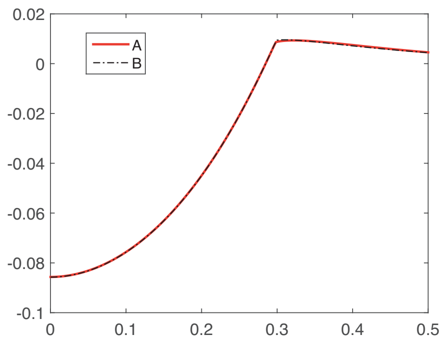
TABLE 1 – Recovered Voce parameters through differrent indentation response ($h_{\max} = 0.1mm$, E and ν fixed).

Case	σ_y (MPa)	$\%err \quad \sigma_y$	σ_s (MPa)	$\%err \quad \sigma_s$	γ	$\%err \quad \gamma$
A	300.21	0.07%	500.44	0.08%	13.90	0.71%
B	260.29	13.24%	461.44	7.71%	41.50	196.43%

Moreover, even not considered in the minimization function of identification, the P-h curves seem to correspond well with each other. Therefore, we conclude that the inverse identification based on indentation is such ill-posed due to an averaged response that even the combination of P-h curve and imprint should not help improve the conditioning of the problem.



(a) P-h curve



(b) Residual imprint

FIGURE 3 – Comparison of indentation responses of different materials.

4 Conclusion and prospective

This work tends to point out the non-uniqueness issue present during the identification of material properties consisted in the Voce law. A so called "mystical material" pair is found by a procedural identification protocol employing the non-linear manifold learning algorithm. The results demonstrates that, the indentation responses, i.e., indentation P-h curve and residual imprint, are averaged interpretations of material constitutive behavior, thus they are incapable of retrieving exactly different material parameters due to independent of response.

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