Variational modeling of pattern formation in thin elastic films under compression

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Elastic thin sheets under compression form a wide range of microstructures. Many microstructures can be undestood as low-energy states of the variational functional given by the theory of nonlinear elasticity, or in term of plate theories. From a mathematical viewpoint, a natural relation arises to the theory of almost-isometric embeddings of two-dimensional manifolds into three-dimensional space.

I shall discuss mathematical progress in the area focusing on specific concrete examples. In the case of a free-standing sheet with a flat reference configuration, such as a crumpled sheet of paper, a complex structure appears, with point singularities connected by ridges. A detailed variational analysis reveals the presence of several length scales in the deformation. The optimal energy for elastic deformations approximating a single fold is proportional to the film thickness to the power 5/3. One important ingredient is the connection to the theory of origami maps, i.e., piecewise affine isometric maps from a subset of the plane to three-dimensional space. In the case of a conical reference configuration the specific symmetry permits a detailed mathematical analysis, which proves the existence of transitions between different response regimes with increasing deformation. For large indentation the optimal deformation recovers the 5/3 energy scaling of crumpled sheets. For films bonded on a substrate more complex patterns appear, with folds on many different length scales.

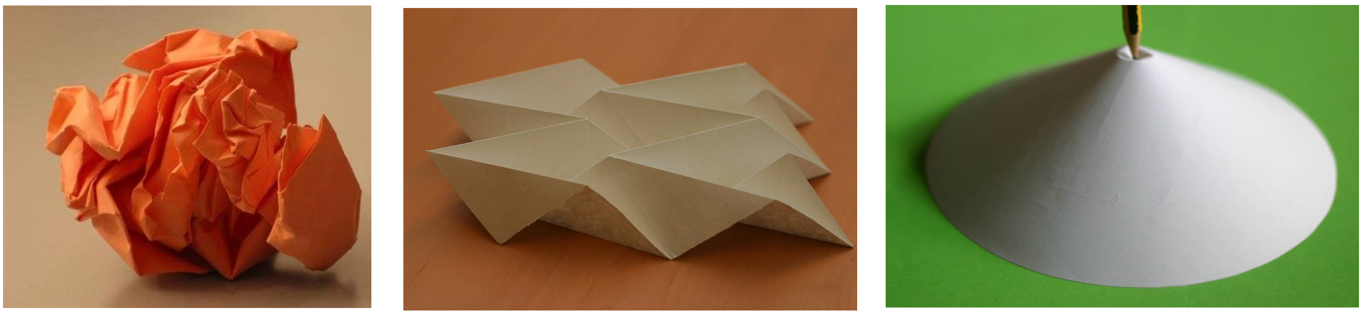

Figure 1. A crumpled sheet of paper exhibits microstructures over many length scales (left). A simpler setting is the one of Origami maps, which are piecewise affine isometries from (a subsets of)) he plane into R^3 (middle). In the case of a conical reference configuration (which may model defects in graphene), then the analysis reveals a spontaneous breaking of the cylindrical symmetry with increasing indentation (right).

References

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